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RECENT DEVELOPMENTS IN VIBRO-TACTILE RESEARCH.*

BY

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I am to discuss recent developments in vibro-tactile research. This, in substance, was the subject matter of my earlier papers (1927 and 1930) before this Institute. By the term "vibro-tactile research," I mean to imply experimental studies that involve, in the first place, the mechanical stimulation of certain sense organs by patterns of vibration in a solid body that is in contact with the skin—in our case with the skin of the finger-tip—and secondly, the ability of human subjects to discriminate among such patterns of stimuli.

The "patterns of vibration," in this case, are those that correspond to speech and to music. These are amazingly complex, as you know, and their variations in respect to intensity alone (or loudness from the angle of hearing) are all but limitless.

When these "patterns of vibration" are brought to the vibro-tactile organs in sufficient intensity we feel them, or, to use a popular phrase—one that is justified even though popular—we "hear through the skin" or "through the fingers" in contact with the vibrating body.

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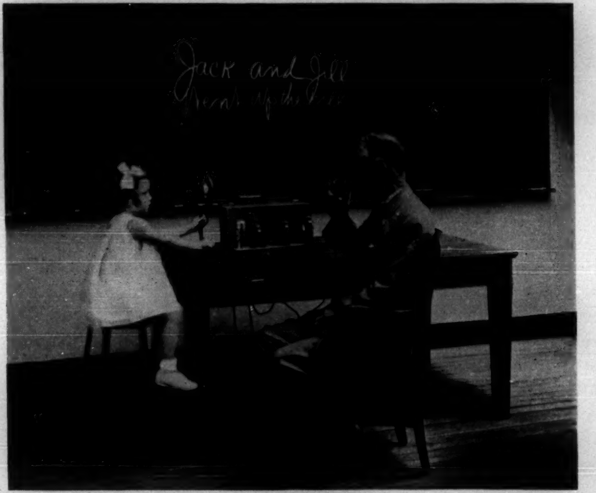
* Presented at the Stated Meeting held Wednesday, December 18, 1935.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

At this moment it will suffice only to mention the fact that our vibrational stimuli must needs be amplified more or less depending upon their frequencies, that is, upon the pitches with which we are dealing. And therein is a very meaty problem in itself. The ideal is so to amplify them that they may be received by the vibro-tactile organs in the same relative degrees of intensity as, in normal conditions, they are received by the ear without artificial amplification.

FIG. 1.



Child receiving speech and using her own microphone; thus feeling her own voice while she attempts to adopt the "model pattern" of her instructor.

The organs that serve as receptors in this situation are those of touch, at least. And I am sure, too, that a sense of vibration is involved likewise, though it is not my purpose, in this connection, to recite my reasons for this conclusion. We have already published a statement on this point.[1] And now you know why I use the hyphenated term "vibro-tactile": because I cannot be sure whether the sense of vibration or that of touch or some other cutaneous or deeper sense is most responsible for the phenomena we are observing, or in what patterns all are together involved. It is probable that one of our laboratory problems, now being investigated, may throw some light upon this question. But to that I shall turn later.

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[1] Goodfellow, Louis D.: "The Sensitivity of Various Areas of the Body to Vibratory Stimuli," *Jour. Gen. Psychol.*, 11, 1934, 435–440.

And now that I have mentioned vibro-tactile "reception" and the phrase "hearing through the skin," I am tempted to repeat what I said, in substance, before this Institute in 1930 and elsewhere at other times: that these sensory responses are a primitive hearing function. Our remote ancestor, the mollusc, became acquainted with his world probably only by touching it. He had literally to scrape up an acquaintanceship with his environment. The hearing of fishes and of other aquatic animals is probably little more than a response of the vibro-tactile system and the same type of reaction may still be a part of the whole auditory function of normal men and women. Dr. Victor Vaughan once said, apropos of these experiments: "We hear, not only with our ears but with our whole bodies." The human ear has evolved from the organs of touch and vibration. Hearing, considered as receiving vibratory stimulation, is an act of touching. And our ultimate practical problem is to determine how far the vibro-tactile organs, with the assistance of appropriate appliances, may be made to substitute for ears.

This phrase, "hearing through the vibro-tactile senses," and what we mean to imply by its use, have become commonplaces among those of us who are daily dealing with the phenomena in question. For sound, from a physical point of view, is vibration. And hearing from any viewpoint, is an act of receiving vibrations on the part of a reacting organism, plus an act of interpreting them. This last is altogether a product of learning. In this compound and complex act of receiving, reacting and interpreting is comprehended the whole of hearing. It matters not at all whether the ear and the auditory nerve or the sense organs in the finger-tip and their neural connections are involved. The whole question is whether vibrations are received, reacted to and interpreted. It is a secondary question—albeit a very important one for practical reasons—what is the measure of completeness of reception, reaction and interpretation, whether the finger or the ear be involved in the process. Whatever the degree of completeness, it is hearing. But it must be said that, at present, we have at best in vibro-tactile phenomena, a very crude and primitive form of hearing.

I shall not make a review of what I have already presented before this Institute on earlier occasions, and of what has been published from my laboratory in various periodicals since 1930. It will be sufficient to recall that in that year I dwelt upon the accuracy of detection of accented syllables, emphasized words and differences in pitch and tempo by means of the vibro-tactile organs, alone, as compared with the accuracy of the same performances by use of the lip-reader's eye alone. Each of these, it will be observed, is a characteristic of spoken language and combines with the others to make up the profile, pattern or melody of speech. Familiarity with this pattern or melody, on the part of both normally hearing and deaf folk, contributes to the interpretation of vocal language and to its successful production. "Contributes to interpretation?" Notice this simple illustration: no two of the following sentences mean the same thing: "It *would* be a great mistake to do that;" "it would be a *great* mistake to do that;" "it would be a great mistake to do *that*." The whole difference is produced by changing the pattern while we leave the words unchanged. When the deaf lip-reader receives the forms of spoken language in his vibro-tactile organs and at the same time observes the speaker's face, he interprets speech around 20 per cent. more fully and accurately than he can do as an unaided lip-reader. This is for the reason that certain characteristics of spoken language, such as emphasis are more successfully detected by the vibro-tactile organs than by the lip-reader's eye alone.

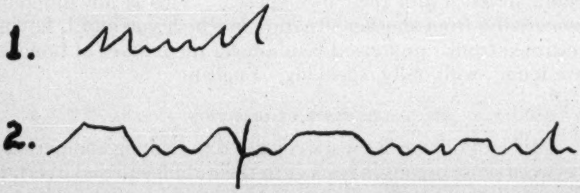
My paper of 1930 before this Institute, as I have said, dealt mainly with this fact. And I am now able to say that since that date I have confirmed this conclusion in iron-clad conditions: conditions in which neither the subject who was receiving the stimuli nor the experimenter who was presenting them was aware either of the preceding experiments or of the aim of the work in which they were participating. It is impossible, therefore, that the results were determined, in the least degree, by unconscious prejudices on the part of either experimenter or subject.

SPEECH PROFILES.

Obvious as all our observations relating to the vibrotactile experience of pattern appear to those of us who are daily immersed in the details of this research, we are frequently confronted with the following question: "But what is it that one receives in the fingers"? Three hearing subjects who were altogether naive in relation to our laboratory have coöperated to furnish an objective answer to this query. Each one was placed alone in a situation that eliminated the possibility of his hearing what was going on. The fingers of one hand rested upon the vibrating unit of the Teletactor. In his free hand he held a pencil, and a pad was in a convenient position. He was instructed that, through the vibrator, he would feel a sentence (the meaning and form of which were not given to him) that had been recorded upon a phonographic disc and that repeatedly actuated the vibrator by way of an electric pick-up and an amplifier, all of which were in a remote room in the laboratory. He was requested to devise a means by use of his pencil and pad for designating the pattern, or profile of this sentence, and of others in succession, as completely as he could manage. No models were before him.

The following figure shows the first and last attempts of a theretofore inexperienced subject to designate the profile of the sentence: "*Now* is the time for *all* good men to aid the

FIG. 2.



party." This is one of eight sentences that were employed in the experiment. From first to last, the subject spent but a half hour upon this sentence. In that time he brought his response to the level I have indicated. He had previously had similar experience with three other sentences of simpler profile. Altogether his earlier work had occupied an aggregate of only ninety minutes, and his performance is typical of that of the two other subjects.

And I do not mean to imply that the second profile includes all the characteristics of the given sentence that the subject really observed. It most probably is not so. It will be obvious that his performance must be restricted as to its completeness and accuracy by the limitations of his pencil in hand—a crude instrument of expression at best in such a situation as I have described. But this type of learning experiment, if so we may describe it, helps us to grasp at least one reason why this type of vibro-tactile experience helps the young deaf child to interpret spoken language and to gain the mastery of his own speech. For there is an analogy between the situation of the subject in this instance, on the one hand, and, on the other hand, that of the very young child in his cradle hearing his mother's voice, or that of the deaf child feeling the voice of his teacher. Both the young hearing child and the deaf child are becoming acquainted with the pattern or profile of normal speech. Therein is his first lesson in spoken language. Now one and then another characteristic of spoken language emerges from the confused background of noise, and in the next instance, both the hearing and the deaf are, wittingly or unwittingly, injecting these characteristics into their own speech. This is not different, essentially, from another situation in which you and I, having returned from a prolonged visit among the natives of London, are found, willy nilly, speaking—English.

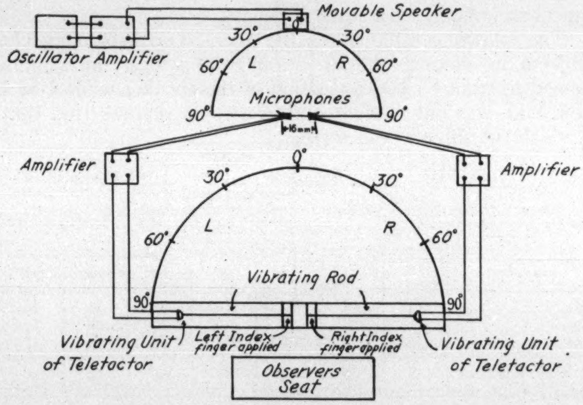
THE SENSE OF DIRECTION.

Much of our earlier work consisted in making comparisons between sense organs in respect to their ability to make certain discriminations. For example, a comparison of the deaf lip-reader's visual and vibro-tactile senses in respect to the detection of a speaker's accent, emphasis, etc.

At this point I shall briefly mention a study of the vibro-tactile senses as means for discovering the direction from which vibratory impressions are coming. This, as you know, is a function that is performed more or less exactly by our auditory senses.

For this purpose we set up our apparatus for vibro-tactile stimulation in such manner as to parallel as accurately as possible the conditions that obtain in binaural localization of the sources of sounds. The following figure will serve to illustrate the arrangement.

FIG. 3.


Movable Speaker
Oscillator
Amplifier
30°
30°
60°
60°
L
R
Microphones
90°
90°
16mm
Amplifier
Amplifier
0°
30°
30°
60°
60°
L
R
Vibrating Rod
90°
90°
Vibrating Unit of Teletactor
Left Index finger applied
Right Index finger applied
Vibrating Unit of Teletactor
Observers Seat

Note :- This apparatus is frequently reversed so as to avoid cues given by inherent differences in the units

Scheme of apparatus for vibro-tactile localization.

It will be observed that the immediate source of vibratory stimuli (the speaker) may be set on the right or the left, 30°, 60° or 90° with reference to two microphones 16 c.m. apart. One of these communicates with the vibrating unit of the Teletactor on the left, and the other with a like instrument on the right. The vibrating units are in a room remote from the speaker and the microphones.

The vibrational stimuli used in this experiment were of three frequencies: 64, 256 and 1024 d.v. Eight subjects or observers participated. It will be obvious that the finger of the right hand resting upon the vibrating rod (Fig. 3) will be stimulated more or less intensively, or with equal intensity as compared with the finger of the left hand in its position, depending upon the location of the speaker. Now when the subject is receiving stimulation in each finger, and when the speaker is moved toward the right or toward the left, he is asked to indicate by pointing to the arc before him the direction from which the vibratory stimuli seem to come. Only when the speaker is in the middle position are the two fingers stimulated with equal intensity.

The following table shows the average error of our eight subjects in judging the direction from which the stimuli seemed to come—i.e. the position of the speaker, which, as I have said, was out of sight and in a room remote from that in which the subject was seated.

TABLE I.

Average Error in Degrees for Eight Observers for Each Position of Speaker. 0 = front median position.

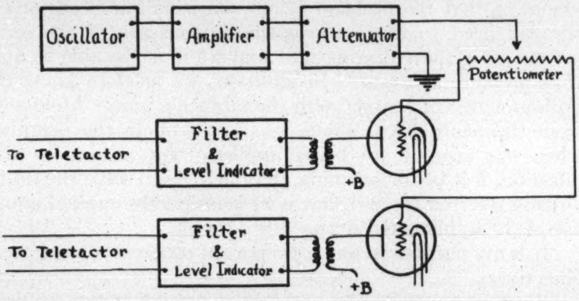
Frequency	0	30L	60L	90L	30R	60R	90R
64	14	19	22	19	18	23	19
256	11	16	18	16	17	20	17
1024	12	14	18	17	15	19	17
Av. error in hearing	6	9	14	11	9	14	10

The figures in the bottom row in the Table indicate the average error in the auditory sphere in a situation closely similar to that described above. It will be observed that, while the errors are unequal in the two spheres they have approximately the same trend right and left from the median.

As I have already pointed out, the situation as I have described it parallels the conditions in normal binaural hearing. For when the source of sound is in the median plane between the two ears they are both stimulated with equal intensity and at the same time. With the removal of the speaker right or left from this plane the ears are stimulated with unequal intensity and one—the farther—a little later than the nearer. It is desirable to isolate the factors of time and intensity.

In the following figure we represent our scheme for varying the intensity of stimulation between the right and the left, while the time factor is maintained constant. The speaker (oscillator) is constantly in the median position (Fig. 3), while the intensity of vibration on the right and left is varied independently; and as the intensity is varied the apparent direction of the source varies also. It seems to the observer much as if an unseen sounding body were moving from right to left or vice versa.

Fig. 4.



Apparatus to vary intensity difference keeping other factors constant.

We have yet to study the effects of quality, phase and time in isolation. The lack of suitable instruments for control has blocked our progress in relation to these factors.

With that I leave my account of our studies of learning by way of vibro-tactile stimulation, and our comparisons of one sensory function with another.

MEANS OF STIMULATION.

It is altogether essential, both for practical and scientific ends that our devices for stimulation be brought to the highest degree of perfection considering the sensory sphere in which lies our major work.

Accordingly, we are moving forward toward two great objectives: first, the improvement of our amplifying and receiving devices to the end, especially, that the essential frequency or pitch characteristics of speech may be received by the vibro-tactile organs as distinctly, and in the same relations of distinctness, one to another, as these characteristics are received by the ear in the usual normal or conventional situation. It is our hypothesis that our human nature can compass anything that is to be learned of the world of sense provided only that it can be brought distinctly into the realm of sensory consciousness. And our second objective is the control of certain conditions that are immediately associated with the stimulation of the sense organs. For example, assuming that the problem of amplification has been solved, we still need to know the optimum pressure of the sense organs upon the vibrating unit and we must be able to hold that pressure constant. In addition, we need to know the optimum area of contact with the vibrating unit. Moreover, from the neurological angle, what goes on in the organism when our stimuli are being applied? An answer to this question, if it be forthcoming, may help us to solve the riddle already referred to, and that is at least partly implied in our use of the hyphenated term—vibro-tactile.

It is my purpose to make a report of progress toward these objectives.

Returning now to the problem of providing the required amplification. We have already published in the JOURNAL of this Institute[2] a statement concerning investigations that have disclosed the differences between the auditory and the vibro-tactile senses in each of several observers in respect to sensitivity to various frequencies of vibrational stimuli, by octaves from 64 d.v. to 8192 d.v. inclusive.

The differences indicated in figure 5 lead us to the amplification that is needed at each frequency. The figure below shows what is required of an ideal amplifier for our purposes and what we have so far been able to accomplish toward realizing it. It will be observed that what is required in our situation is quite different from what is demanded of a radio receiving set.

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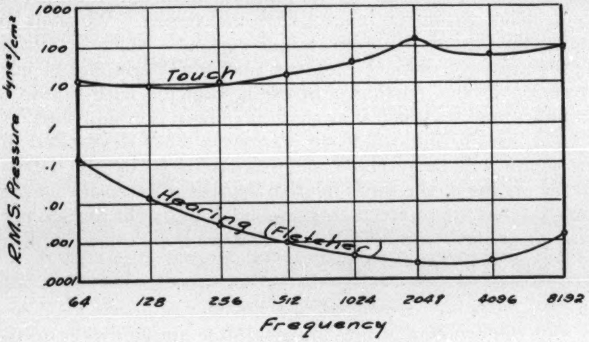
[2] Goodfellow, Louis D.: "The Sensitivity of the Finger-Tip to Vibrations of various Frequency Levels," JOUR. FRANK. INST., 216, 1933, 387–392.

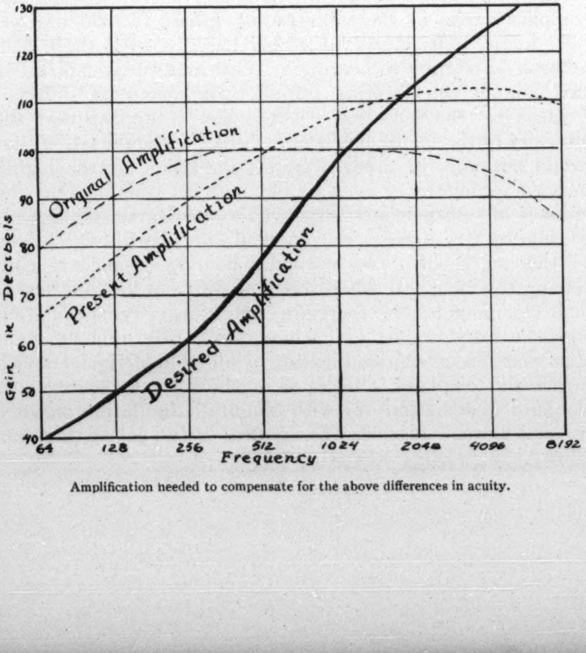
FIG. 5.



Relative vibro-tactile and auditory acuity for pitches.

At this juncture we have the engineering aspect of our research. It is obvious from Fig. 6 that we have yet to give

FIG. 6.



Amplification needed to compensate for the above differences in acuity.

considerable attention to the amplification of the higher frequencies within the range I have mentioned. We want to make the curve "present amplification," which in itself represents very distinct progress since the beginning of our attention to this particular problem, coincide with "desired amplification." When that shall have been accomplished, as I have already indicated, the various pitches designated in the figure will be received and responded to by the vibro-tactile organs in the same relative degrees of intensity as the same pitches are received and responded to by the normal ear without artificial amplification. This we hope to accomplish by means of a series of filters or otherwise.

THE PROBLEM OF MASKING.

But the problem of providing suitable amplification up to a certain frequency limit is fraught with complications incident to interference when vibrational stimuli of more than one frequency are simultaneously activating the vibrating unit and thus increasing the complexity of the stimulus upon the sense organ. Interference of this order in the auditory sphere results in the phenomena of "masking." How serious this complication is in the vibro-tactile sphere, we do not yet fully know. Dr. Harvey Fletcher has discussed these phenomena in relation to hearing.[3] When one ear is exposed to two sources of vibrating impulses of somewhat different frequencies, the sensation effect makes it appear that the intensity of the higher has been reduced. The sensation that would normally be aroused from stimulation by the higher frequency alone may even be eliminated assuming that the lower is of sufficient intensity. To a less degree the lower is masked by the upper. No practical difficulty is experienced on this score when two normally hearing people are conversing together and when tones of different frequencies or their compounds are therefore simultaneously stimulating both auditory organs of each person. But difficulties do arise when we are communicating by aid of an artificial device such as the telephone, a situation in which we are dealing with monaural stimulation—or with binaural stimulation on very unequal terms. Persons who are deaf in one ear are familiar with the confusion that is incident to their listening to two or more people who are simultaneously talking. Therein is an exaggeration of some of our difficulty at the telephone. It will be easily understood that if, in the auditory sphere, masking were so complete as to bring all overtones below the intensity threshold for hearing, the remaining fundamentals will not present sufficiently different patterns of stimulation to enable us to discriminate, as we normally do, among the characteristics of speech. The analogy between the vibro-tactile and auditory spheres suggests the desirability of our investigating masking phenomena in the former.

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[3] "Speech and Hearing," New York, 1929, pp. 167–180.

It will be understood, therefore, that this investigation may throw even more light than we now have—see the last figure—upon the question what an amplifier must do in order that we may advance further toward discovering the conditions in which the vibro-tactile organs will contribute their maximum toward the improvement of speech and toward its interpretation. Our preliminary experiments have afforded satisfactory proof that the sensory response to a single tonal stimulus—128 d.v., e.g.—can be distinguished from the response to such a double stimulation as 128 d.v. and 1024 d.v., to take but one example among many, when these two are applied simultaneously. But the intensity of the higher tone at its source had to be increased by 10 d.b. to make the distinction possible. In other words, the masking effect of 128 d.v., in this instance, appears to be equivalent to 10 decibels or units of intensity.

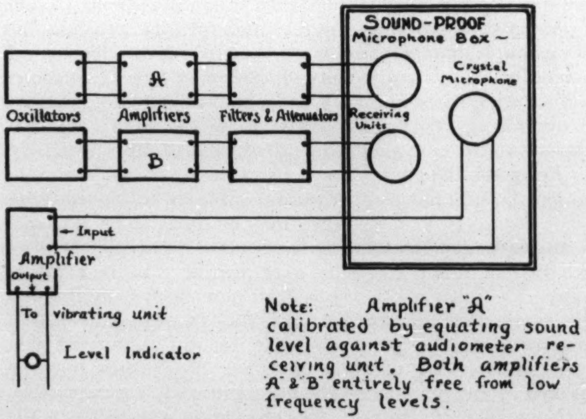
From the data we have at hand, however, we cannot at present be sure but that, when our subjects distinguish "one tone" from "two tones," they may be doing so on the basis of intensity alone and not on the basis of tonal quality, or of two distinct qualities of tone, as in hearing. In the auditory sphere, the inner ear or the central nervous system analyzes the compound and we are conscious of two pitches that correspond to the vibratory rates 128 d.v. and 1024 d.v. respectively. One of these products of analysis may be masked by the presence of the other but each is recognizable.

Now again in respect to masking in the vibro-tactile sphere. As far as we are able to know and understand the sense organs involved, there is no possibility of analysis in or by the organs themselves. If, therefore, in the future course of these experiments, our subjects are still able to distinguish two qualities when two stimulus patterns are present; and if they can do this notwithstanding that the patterns are of equal intensity though their component pitches vary, we shall thereby contribute not only to the subject of masking in the vibro-tactile sphere, but we shall also strengthen the hypothesis that the analysis of compounds in the auditory sphere is not a sense organ but a central function.

At this point our investigation of the problem of "masking" has been temporarily halted. We are not, however, without a plan of procedure.

Hitherto, in the course of our preliminary experiments, the vibrational patterns that we were attempting to communicate were electrically combined at their source. For the future, our plan is to combine them acoustically in the air enclosed in a sound proof box. Within the box are two receivers, each connected with its own source through a suitable amplifying device.

FIG. 7.


SOUND-PROOF
Microphone Box ~
Crystal Microphone
A
Oscillators
Amplifiers
Filters & Attenuators
Receiving Units
B
← Input
Amplifier
Output
To vibrating unit
Level Indicator
Note: Amplifier "A" calibrated by equating sound level against audiometer receiving unit. Both amplifiers "A" & "B" entirely free from low frequency levels.

Schematic arrangement of apparatus for investigating vibro-tactile masking.

The patterns thus combined within the box are then taken up by a microphone, and transmitted by way of another amplifier to the vibrating unit of the Teletactor and thus to the vibro-tactile organs.

If masking in this sphere should prove to be very pronounced, and if it cannot be obviated by looking only to the amplifying unit of the Teletactor, it may become necessary for us, in the interest of the best practical results, to reconsider the design of the receiving or vibrating unit.

This is, in its present form, a single magnetic unit and its vibrations are brought to bear, in practice, upon a single finger-tip; a situation that suggests an analogy to monaural as opposed to binaural hearing. It will be recalled that masking is especially embarrassing in the auditory sphere when a single ear is stimulated. Our experiments with this phenomenon in the vibro-tactile sphere may lead us to introduce filters and a multiple unit as a substitute for our single unit receiver, through which the whole range of frequencies would be divided into a convenient number of bands—even as many as five—each band operating a separate vibrating unit, no two of them in contact with the same area of skin. Such a set-up—as far as masking is concerned—has the following to recommend it: it will minimize whatever masking effect there may be. Moreover, it will introduce an additional criterion in connection with the discrimination of forms of speech: that is, cutaneous localization of stimulation corresponding to different frequencies.

SENSE ORGAN AND NEURAL FUNCTION.

But assuming that problems connected with amplification and with interference or masking are out of the way, there remains the second great objective of our present activities to which I have already referred. This involves several problems of major importance. They are concerned with the functioning of the sense organs and less directly with the management of the stimuli. The first of these is suggested by the question: What is the optimum resistance or pressure between the vibrating unit and the vibro-tactile organs? The second, what is the optimum area of stimulation? And the third problem is designed to contribute toward an answer to the question how nerve processes—action currents—correspond to a vibratory stimulus applied to the skin in the absence of hearing.

In relation to the effect of pressure, it will be obvious that any addition thereof above a given value will tend to damp the response of the vibrating unit, and that will result in reducing the intensity value of the vibro-tactile sensation, even to the vanishing point. It is extremely important, therefore, to know the optimum pressure and, having found it, to be able to control it, not only for research purposes in the laboratory but for teaching purposes, also, in the rough and tumble of the school room.

Moreover, apart from the damping effect of pressure upon the vibrating unit, there is its effect upon the response of the sense organs themselves. It is desirable, both for theoretical and practical purposes, that on this account, too, the optimum pressure be found and maintained constant. But our experiments in this relation, and in respect to the optimal area also, are in such a preliminary stage that it is unprofitable to discuss them now.

Our third problem must necessarily be investigated by experimentation upon lower animals—the dog for example. Assume that an animal whose vibro-tactile and auditory mechanisms are intact has established conditioned reactions to vibratory stimuli, cutaneously applied. Does he retain these reactions after sectioning the eighth nerve? Can he establish new conditioned reactions after such an operation? Sever the posterior spinal cord tract that carries impulses from vibratory stimulation. Does such an animal, otherwise intact, retain previously established conditioned reactions? Can he establish new reactions of a similar nature? Compare his action currents with those that have been observed in other situations.

This is an elaborate and very complicated program, some aspects of which have hitherto been baffling.

We are fortunate in having the active coöperation of the Department of Physiology in the Northwestern University School of Medicine in this connection.

Only such an intensive study of the possibilities of establishing and retaining conditioned reactions in these instances; a study of the forms of action currents in relation to a given stimulus pattern, when the only altered conditions are those involved in severing the eighth (auditory) nerve and the spinal cord tract respectively: only such studies can unravel for us the relations of the senses of hearing, of touch or light pressure and of the sense of vibration, the distinctive character of which is, in some minds, questionable.

Our intermittent preliminary work on this complicated problem during the last two years has only cleared the ground and shown us what appears to be the best line of advance.

We attempted a year ago to rest the whole case upon conditioned reaction experiments employing human subjects who are victims of tabes dorsalis: a condition in which the sense of vibration is lacking due to a deterioration of the posterior tract in the spinal cord. But this procedure proved to be impracticable for the reason that we did not succeed in finding cases in which a post mortem examination demonstrated beyond the shadow of a doubt that deterioration of the tract had proceeded to the point of its complete interruption. We have therefore in the last months resorted to our friend, the dog, and to surgical technique as a means for producing the situation that the experiment requires.

I have now completed my survey of recent progress in vibro-tactile research in my own laboratory. I need hardly draw attention to so patent a fact as that our program is one that requires years for its completion.

In conclusion: since I last had the honor to appear on the program of this Institute we have made the following definite accomplishments:

1. Our earlier results showing that the eye and the vibro-tactile organs may be made to coöperate advantageously in the management of spoken language have been unequivocally confirmed.

2. We have added the following to our earlier reported parallels between the vibro-tactile and other sensory functions: they both, in much the same manner, furnish clues as to the apparent direction of vibrating bodies.

3. We have found the differences in sensitivity between the auditory and the vibro-tactile spheres for various pitches. Thus we have discovered the goal to be reached in constructing a suitable amplifier for vibro-tactile stimulation.

4. Preliminary experiments have cleared the way for the study of masking in the vibro-tactile sphere; for finding the optimum pressure and area of stimulation and for observing the nature of neural processes under vibratory stimulation.

Chemical Glassware Sans Arsenic.—Apparently arsenic is suitable for clearing up things other than human complexions. In the batch or mixing room of almost any glass plant, posted directions will call for the addition of certain quantities of arsenic trioxide. This product aids materially in producing a clear glass free from small bubbles or seeds. Now, in the *I. and E. C.*, News Edition, Vol. 14, p. 23, W. C. TAYLOR announces that no longer will arsenic be used in Pyrex glass as a clarifying or fining agent.

Even when a few tenths per cent. were used formerly, tests showed that no arsenic was extracted from this glass under ordinary working conditions owing to the extreme resistance of the glass to chemical attack. The elimination of the arsenic was accomplished by gradual reduction until now analyses of Pyrex show only 0.1 to 0.3 parts per million of arsenic trioxide. Its practical absence of arsenic has imparted a greener color to the glass, best observed when viewed end on in the form of a tube or rod. The color of the flame at softening temperatures also is different and the glassblower must readjust his working to the new appearance.

An incidental but significant advantage also resulted from the arsenic elimination, namely, higher ultraviolet transmission. In a thickness of 2 mm. instead of about 10 per cent. transmission at 3020 Å. with a transmission limit of 2900 Å., we now have 50 per cent. transmission at 3020 Å. with a limit of about 2700 Å.

C.

Age Cannot Wither Her . . . .—At a joint annual meeting of more than five hundred geologists, mineralogists and paleontologists, WILLIAM D. URRY explained his work in measuring the life of rocks by their radioactive decay. By applying the latest methods of microchemical analysis, Dr. Urry determines how much of the radioactive elements is still intact in the rock and how much has decayed, as indicated by the quantities of lead and helium which were the result.

The rocks of the Hudson River Palisades are 150,000,000 years old, rocks near Duluth, Minn., were found to be more than 500,000,000 years old, while rock specimens from other parts of the world showed an age of 900,000,000 years. According to ALFRED C. LANE, chairman of a special committee on the measurement of geologic time, these determinations indicated the age of the earth as a planet as not less than two billion years.

C.

MONOCHROME REPRODUCTION IN PHOTO-ENGRAVING.*

BY

ALEXANDER MURRAY.

I. INTRODUCTION.

The work reported here was begun as a subdivision of some research on the reproduction of colored originals by three- and four-color press-printing processes.

A purely photographic process of multi-color plate-making could not be realized unless it gave a practically perfect monochrome reproduction of the density scale on each plate. Since the literature and personal experience indicated that this had not yet been accomplished, it became necessary to investigate it. For the intended purpose, it was required to determine whether or not perfect reproduction of a density scale was experimentally possible in the half-tone processes and, if it were possible, whether or not it could be consistently repeated. The scope of this work was limited to the formulation of one dependable routine for producing perfect plates without the intervention of manual methods of correction. The information was needed for making plates and proofs of color experiments. The proposed application demanded that the photographic steps be indirect; that is, the first negatives should be continuous tone, and from these the half-tone screen positives or negatives should be made later.

In this connection, relief photo-engraving was considered more suitable than offset lithography or photogravure for several reasons. It is most commonly used commercially. The plate-making was estimated to have fewer variables and to be less expensive as an experimental method than photogravure. Once the plate was made, printing was expected to be more mechanical and less liable to human variation than either offset or gravure. Reproduction curves already published for the photogravure process [1] were interpreted as being less encouraging than some obtained for photo-engraving.[2] Under optimum conditions, monochrome proofs from photo-engravings show higher maximum density than gravure and offset proofs. Tones are less granular in photo-engraving than in offset. Accordingly, while the author was fully aware of the merits of the other processes, relief photo-engraving was selected as best adapted to the purpose in view in the present state of the art.

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* Communication No. 585 from the Research Laboratories, Eastman Kodak Company, Rochester, New York.

II. CRITERIA.

A point to be considered is whether the 45° straight-line reproduction curve on a logarithmic density graph, applied without qualification to halftone screen images by A. J. Bull,[2] is a complete or appropriate criterion of halftone reproduction.

H. E. Ives [3] applied L. A. Jones' graphic method [4] in a theoretical examination of halftone reproduction. Jones [4] and M. Abribat [5] have emphasized that the straight-line final curve, either in Jones' method or on a simple density-density graph, can only be strictly applied when the reproduction is viewed under the same peripheral conditions and at the same adaptation level as the original. If a sketch is mounted on a black card and the reproduction is printed with a margin of white paper, the peripheral fields are different, and an ideal reproduction curve will depart from a theoretical 45° straight line. In the present instance, the original is a "picture," which may be a photograph, painting, or drawing, that can be compared directly with the photo-engraver's proof. The assumption is, therefore, justified that such originals can be mounted upon or viewed against a field of approximately the same character as that surrounding the reproduction for purposes of criticism, and that the two can be compared simultaneously in approximately identical illumination.

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[1] H. Mills Cartwright, "Experiments Relating to the Rendering of Gradation in Photogravure," *Phot. J.*, Dec. (1921), 428–434.

[2] A. J. Bull, "Tone Rendering by Half-Tone Processes," *Proc. 7th Ann. Congr. Phot.* (1928), 374–384.

[3] H. E. Ives, "Tone Reproduction in the 'Half-Tone' Photo-Engraving Process," *J. Opt. Soc. Am.*, *13* (1926), 537–552.

[4] L. A. Jones, "On the Theory of Tone Reproduction, with a Graphic Method for the Solution of Problems," J. FRANK. INST., *190* (1920), 39.

[5] M. Abribat, "General Considerations on the Photographic Reproduction of Luminosities," *Sci. et Ind. Photographique*, 2, *6* (1935), 177.

By confining the problem to the above specifications which seem practically reasonable, it is possible to avoid the difficulty of a reproduction curve whose character would have to change with every variation of the respective viewing conditions of original and reproduction.

A question has also been raised [6] as to the agreement of curves based upon scales of (*a*) steps of equal log $1/R$ reflection density increments, and (*b*) equal subjective "value" steps such as might, for example, be estimated by an artist. In reproducing a picture, this problem can be ignored. The scale of densities photographed as a control with the original need not be a scale of equal or regular steps in any sense, because a point-for-point reproduction of such a scale will give a straight-line reproduction curve when densities are plotted against densities regardless of irregularities in the original scale, and both picture and scale will be correctly reproduced, provided the maximum and minimum densities of the scale correspond with those of the picture.

As far as the writer knows, the first noteworthy attempt to obtain exact information on reproduction in photo-engraving was a laborious series of experiments published by N. S. Amstutz in 1908 [7] and 1910.[8] This author's standard of reference was a straight-line curve on a linear plot. He calculated for each density step the per cent. area of black ink based upon microscopic measurements of dots, and plotted these against assumed regular 5 per cent. reflection increments in the original. Measurements made by the writer from reproductions of step wedges illustrating his paper do not indicate very close correlation between his calculated black areas and the reflection densities measured. The probable transmission densities of his celluloid sheet "actinometer" can be roughly estimated, but his actual test object was a toned print from this on printing-out paper. The H. and D.

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[6] M. Abribat, private communication.

[7] N. S. Amstutz, "A Comparison of Metzograph and Half-Tone Interpreting Values," *The Graphic Arts and Crafts Yr. Bk.* (1908), 55–73.

[8] N. S. Amstutz, "Three-Color Reproduction Susceptibilities," *The Graphic Arts and Crafts Yr. Bk.* (1910), 99–126.

characteristic curve of printing-out paper has a long, gently rising toe region, which resulted in curves that, when replotted from the reflection densities of ink impressions of Amstutz's experiments against estimated transmission densities of his actinometer, were nearly horizontal from highlight to middle tones.

One thing about Amstutz's work will be emphasized here. When he drew his reproduction curves he marked the 100 per cent. line, "Dead Black," and base line, "Pure White." He explained that "Dead Black" is solid metal on the printing plate, unbroken by halftone dots, while "Pure White" contains no printing areas whatever. The significance of this, although again mentioned by H. E. Ives in 1926,[3] has not always been kept in view by others working on the problem. It leads to a special criterion for reproduction in halftone plates.

A halftone print is a structured image in which variations in density are rendered by variations in the ratio between areas of solid ink and areas of blank paper. On a paper proof in black ink the scale of densities is limited by the reflecting power of the paper itself at the highlight end, and the density of unbroken black ink at the shadow end. The density of solid black ink impressions from photo-engravings varies, commercially, from about 0.8 to 1.6, or even higher, due to variations in blackness of ink and thickness of the printed film. The halftone structured image simply divides the scale between these limits into intermediate densities. That is its only function, and it should be obvious that it is a fallacy to expect halftone plates ever to yield denser shadows than the density of the ink used for printing, or lighter whites than the paper stock used.

Since the scale of densities in a halftone printing plate is purely a structural differentiation between the two limits of "solid" and "blank," it should be possible to arrive at density values by measuring dots under a microscope and calculating their areas. To indicate the possibilities of this method, a 150-line copper photo-engraving was used [9] which, printed in red ink, gave a straight-line reproduction curve when log $1/R$

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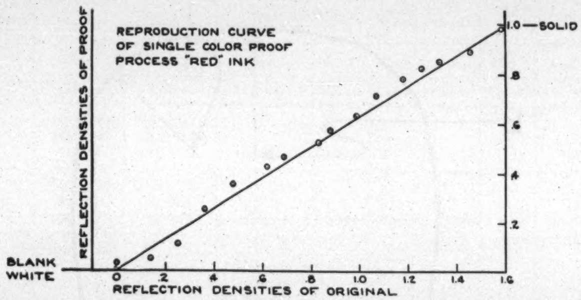
[9] The preparation of this plate will be fully described in a future paper.

densities, measured through a complementary filter,[10] were plotted against original densities, as in Fig. 1.

This plate appeared ideal for testing the micrometric method. It was clamped on the stage of a Leitz microscope, equipped with an "Ultropak" illuminator and an ocular with a micrometric scale which was calibrated before use. The calibrated scale units were equivalent to 0.00206 mm. at a magnification of approximately 200 diameters.

The screen pitch, or distance between two adjacent dots from center to center in the direction parallel to the screen ruling, was measured for 12 pairs of dots, and averaged. The theoretical pitch is 0.1693 mm. The measured value was 0.1732 mm. which gives an area of 0.0300 mm.^{2} for one whole screen unit. For each density step of the gray scale, three dots were measured—one in the middle and one near each edge. These values were also averaged. Since the dots are not all perfectly round, some being more or less flattened on four sides corresponding to the square screen opening, their dimensions were measured on both axes, as in Fig. 2. Such a dot was calculated as a circle with a diameter equal to the

FIG. 1.



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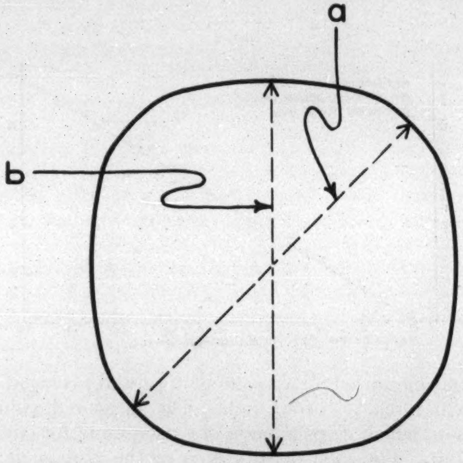
[10] The Eastman Transmission and Reflection densitometer is provided with a filter ring, by means of which appropriate gelatin film filters may be interposed in the optical path in the eyepiece. For reading densities of process "red" ink proofs and chalked-up copper plates, a Wratten No. 59 filter is satisfactory. For "blue" and "yellow" proofs, Nos. 25 and 47 filters are used, respectively.

mean of the two axes. It will appear from the results that this abbreviated method, although crude, has sufficient accuracy for the intended purpose. In the highlights, the "black" dots were measured, but in the regions where the black dots are completely joined at the corners, the white dots were measured because they were approximately circular in shape. At the fifth step in the scale, the black dot was calculated as a square measured on its lesser axis, and the white dot was calculated as a circle measured on its greater axis. The difference in the areas was split to arrive at the estimated area of the black dot. The data are given in Table I.

FIG. 2.



If the fractional areas in Column 5 are regarded as equivalent to densities and plotted against the original densities, the curve, "*B*," Fig. 3, is seen to agree closely with the curve of an ink proof of $D_{\max.}$ 0.98 measured on the densitometer. This is a contradictory agreement and purely accidental. It is an unusual coincidence. Per cent. black area has no linear connection with density and the two cannot be compared directly;

nevertheless, it is evident that dot structure must be some function of density.

The writer is indebted to E. R. Davies [11] for a simplified formula correlating density and structure. Theoretically,

TABLE I.

Micrometric Data.

Comparison of 150-line copper photo-engraving with proof on enamel paper in black ink of $D_{\text{max.}} = 1.46$.

Unit Areas: 0.0300 mm.^{2} for plate, 0.02931 mm.^{2} for proof.

Step Nos.	Linear Measurements Average of Six, in Mms.		Calculated for Printing Areas, or "Black" Dots.					
			Area, in Mm.^{2}		Fraction of Unit Area.		Per Cent. of Unit Area.	
	Plate.	Proof.	Plate.	Proof.	Plate.	Proof.	Plate.	Proof.
	Col. 1.	Col. 2.	Col. 3.	Col. 4.	Col. 5.	Col. 6.	Col. 7.	Col. 8.
1	Black Dot Radius .00360	Black Dot Radius .02710	.0000407	.00231	.00130	.0788	0.13	7.88
2	Black Dot Radius .02110	Black Dot Radius .03370	.001399	.00341	.04663	.1163	4.66	11.63
3	Black Dot Radius .03195		[Mean of .003217 .004083]					
	Side of Square .06390	Black Dot Radius .04335	.00365	.00590	.12167	.2013	12.17	20.13
4	Black Dot Side of Square .08650							
	White Dot Greater Axis .07725	Black Dot Side of Square .05285	Mean Area .00810	.01120	.27000	.3821	27.00	38.21
5	White Dot Radius .07411	White Dot Radius .06540	.01273	.01587	.42433	.5539	42.43	55.39
6	WhiteDot Radius .06955	White Dot Radius .05985	.01480	.01806	.49333	.6122	49.33	61.22

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[11] E. R. Davies, private communication.

TABLE I.—*Continued.*

Step Nos.	Linear Measurements Average of Six, in Mms.		Calculated for Printing Areas, or "Black" Dots.					
			Area, in Mm.^{2}		Fraction of Unit Area.		Per Cent. of Unit Area.	
	Plate.	Proof.	Plate.	Proof.	Plate.	Proof.	Plate.	Proof.
	Col. 1.	Col. 2.	Col. 3.	Col. 4.	Col. 5.	Col. 6.	Col. 7.	Col. 8.
7	White Dot Radius .06540	White Dot Radius .05900	.01656	.01837	.55200	.6267	55.20	62.67
8	White Dot Radius .06130	White Dot Radius .05110	.01819	.02111	.60633	.7201	60.63	72.01
9	White Dot Radius .05975	White Dot Radius .04770	.01878	.02216	.62600	.7560	62.60	75.60
10	WhiteDot Radius .05410	White Dot Radius .03940	.020905	.02443	.6970	.8335	69.70	83.35
11	White Dot Radius .04995	White Dot Radius .03535	.022162	.02538	.73873	.8659	73.87	86.59
12	White Dot Radius .04530	White Dot Radius .03610	.023553	.02520	.78510	.8598	78.51	85.98
13	White Dot Radius .04170	White Dot Radius .02420	.024537	.02747	.81790	.9372	81.79	93.72
14	White Dot Radius .03555	White Dot Radius .02847	.026030	.02847	.86767	.9713	86.77	97.13
15	White Dot Radius .03400	White Dot Radius .00630	.026368	.02929	.87893	.9959	87.89	99.59
16	White Dot Radius .01495	White Dot Radius 0	.029298	.02931	.97660	1.0000	97.66	100.00

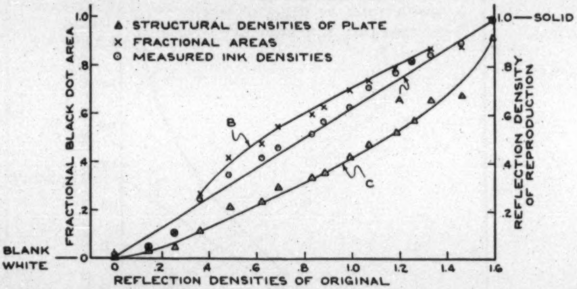
provided density distribution is uniform and the printing density of the ink is known, structure can be calculated from density, and vice versa. If D = density, a = fraction of unit screen area occupied by the printing surface, which is here referred to as black dot area, and r = reflecting power of the ink, then

$$D = \log \frac{1}{1 - a(1 - r)}.$$

The most cursory microscopic examination discloses the fact that density distribution in a single printed dot is anything but uniform, but the hypothetical assumption will be adopted at this point that in each dot the *average* density is identical with the density of a large unstructured ink area. There is experimental evidence favoring this assumption.

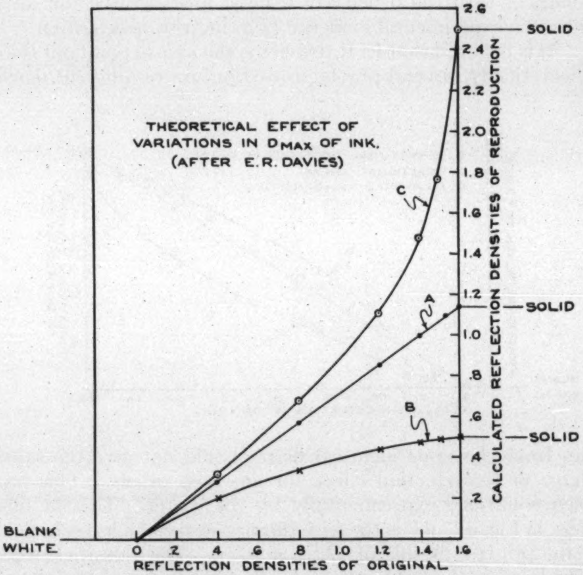
It is believed that E. R. Davies is the first to point out that, theoretically, several proofs printed in inks of different densities from the same identical plate should not give the same shape of reproduction curve for any two proofs. This has been confirmed experimentally by the writer. To take one case, in Fig. 4, "*A*" is the reproduction curve of a hypothetical plate printed in ink of $D_{\max.} = 1.14$. The densities were calculated back to structure, and then recalculated for inks of $D_{\max.} = 0.5$, and 2.5, marked "*B*" and "*C*," respectively. The figure shows that a variation in both shape and gamma can be expected. Figure 5 shows a series of actual curves obtained from a single plate. "*A*" consists of densities measured on the chalked-up engraving, giving a density of 1.14 for a structureless solid. "*B*" was measured with a densitometer from a proof in black ink of $D_{\max.} = 1.44$. "*C*" is from a

FIG. 3.



heavier proof of the same ink. "*D*" is from a proof in the same black ink reduced with colorless base to $D_{\text{max.}} = 0.48$. "*E*" is from a proof in process yellow ink measured through a Wratten No. 47 filter. Curve "*C*" has developed a marked shoulder, which is explained later. Figure 5 confirms the type of variation forecast in Fig. 4. Of course, no complete criterion of halftone reproduction can ignore this effect.

FIG. 4.



Davies' formula has been used to calculate densities from the micrometric data in Table I. The first calculation was for an ink of $D_{\text{max.}} = 1.0$. The resulting densities are listed in Column 2, Table II, and were plotted as curve "*C*," Fig. 3, in comparison with actual printed densities in ink of $D_{\text{max.}} = 0.98$, curve "*A*." Curve "*C*" fell far below "*A*." Thus in Fig. 3 there was a close agreement between curves "*A*" and "*B*," which should disagree, and disagreement between "*A*" and "*C*," which might have been expected to agree. Evidently, more information was required to correlate the latter.

Proofs were pulled from the same plate in ink of $D_{\text{max.}} = 1.46$. Microscopic measurements made on one of these proofs are recorded in Table I beside those for the copper plate. Densities were calculated from the micrometric data of a proof, for comparison with actual densitometer readings of three proofs, which were averaged. These two sets of densities are given in Columns 3 and 4, Table II. They appear in Fig. 6, plotted against original densities. This time the agreement was good in a rough way. Although the proofs were printed on smooth enamel paper, the printed dots in any one tone varied so in size and shape that only a rough estimate

FIG. 5.

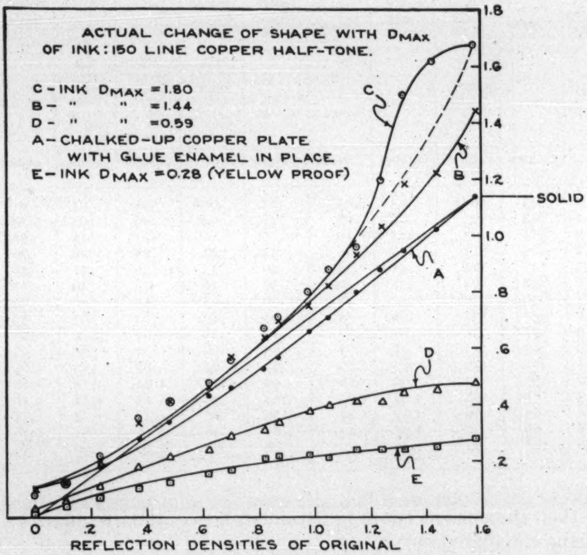


TABLE II.

Measured and Calculated Densities.

Densities Measured on Eastman Transmission and Reflection Densitometer. All derived from the 150-line plate whose dimensions are given in Table I.

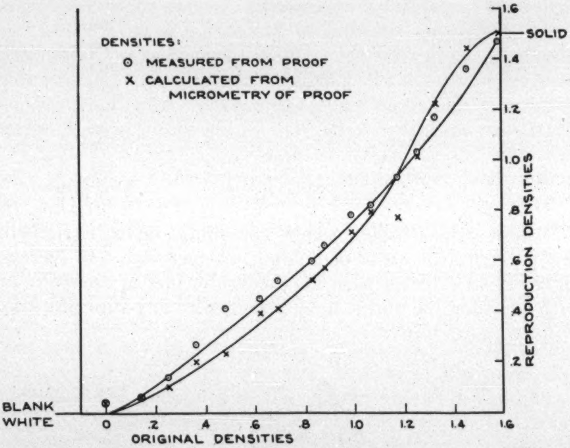
Step Nos.	Log. 1/R Densities.								
	Proof in Process Red Ink Through No. 59 Filter $D_{\text{max.}}$ = 0.98.	Calculated from Plate for Ink of $D_{\text{max.}}$ = 1.0.	Proof in Black Ink Average of three: $D_{\text{max.}}$ = 1.46.	Calculated from Micrometry of Proof $D_{\text{max.}}$ = 1.5.	Proof in Black Ink Varnished $D_{\text{max.}}$ = 1.88.	Calculated from Plate for Ink of $D_{\text{max.}}$ = 1.0.	Calculated from Plate for Ink of $D_{\text{max.}}$ = 1.5.	Calculated from Plate for Ink of $D_{\text{max.}}$ = 1.9.	Densities of Original Gray Scale.
		Spreading Increment 0.				Ink Spread Increment .01 + .0005 × $(n - 1)$.	Ink Spread Increment .01 + .001 × $(n - 1)$.	Ink Spread Increment .01 + .001 × $(n - 1)$.	
	Column 1.	Column 2.	Column 3.	Column 4.	Column 5.	Column 6.	Column 7.	Column 8.	Column 9.
1	.03	.005	.03	.035	.03	.008	.01	.01	0
2	.05	.03	.06	.05	.06	.04	.05	.05	.14
3	.11	.05	.14	.10	.15	.10	.11	.11	.25
4	.25	.12	.27	.20	.32	.24	.28	.29	.36
5	.35	.21	.41	.23	.49	.33	.40	.41	.48
6	.42	.24	.47	.39	.59	.38	.48	.49	.62
7	.46	.30	.52	.41	.66	.45	.55	.58	.69
8	.52	.34	.60	.52	.73	.49	.64	.67	.83
9	.57	.36	.67	.57	.78	.53	.68	.71	.88
10	.63	.43	.78	.71	.85	.60	.81	.85	.99
11	.71	.48	.83	.79	.97	.67	.92	.99	1.07
12	.77	.53	.93	.78(?)	1.09	.73	1.04	1.13	1.18
13	.82	.58	1.04	1.03	1.24	.79	1.15	1.28	1.26
14	.85	.66	1.17	1.22	1.39	.87	1.32	1.54	1.33
15	.89	.68	1.35	1.44	1.61	.90	1.38	1.64	1.46
16	.98	.92	1.46	1.50	1.88	1.00	1.50	1.90	1.59

of their dimensions could be attained; hence, the irregularities in the calculated curve.

Referring to Table I, a comparison of dimensions of plate and proof discloses a gain in area for the dots in the proof at every step. This is due to spreading of the ink beyond the edges of the dot. Practically, this spreading is a consistent accompaniment of relief printing. When the plate is inked, some ink is rolled over the edge of the dot and, during the impression, the ink is compressed between metal and paper surfaces, resulting in more ink being squeezed out at the edges.

Amstutz[12] assumed that the increase in linear dimensions, due to spreading, was approximately uniform over the entire scale. He cited an example in which the spreading was assumed to be 0.0127 mm. on the dot radii for a 100-line screen, and calculated the effect upon per cent. black areas. His conclusion that darkening due to spreading was greatest in the highlights is not justified by his own data, which show a small dimensional change at each end of the structural per cent. scale, becoming larger toward the middle of the scale. Compare also Columns 7 and 8, Table I, in the present paper.

FIG. 6.



When *density* is considered, the effect of spreading, although small dimensionally, becomes very large, especially in the deeper shadows. The amount of spreading, of course, can be obtained directly by subtracting the micrometric plate dimensions from those of the proof. This method was used to obtain the rather erratic values of Column 1, Table III. Spreading is expressed here as difference in the radius of a circular dot.

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[12] N. S. Amstutz, "Hand-Book of Photo-Engraving" (1913), 311–314.

TABLE III.

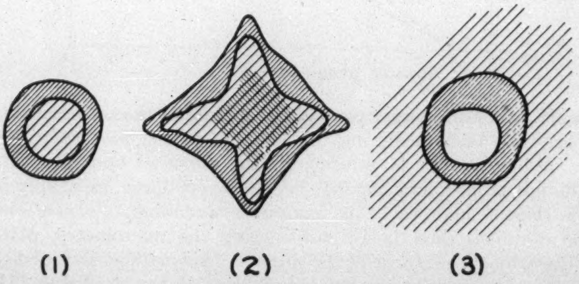
Comparison of Ink Spreading and Ink Ring.

150-line photo-engraving, and black proof on enamel paper.

Step Nos.	Ink Spreading: Calculated as Difference between Dimensions of Plate and Proof.	Ink Ring: Average Width Measured Microscopically.
1	.0235 mm.	.010 mm.
2	.0126	.008
3	——	.010
4	——	.010
5	.009	.010
6	.010	.015
7	.0064	.015
8	.0102	.020
9	.015	.020
10	.015	.024
11	.015	.025
12	.009	.020
13	.017	.030
14	.019	.026
15	.028	.025
16	Structureless	Structureless

These values indicate that spreading, on this particular proof, was of the order of .01 mm. to .025 mm. It averaged higher for the denser than for the lighter half of the scale, but with considerable uncertainty attached to any interpretation.

FIG. 7.



Halftone dots, printed from relief plates on smooth stock, show a characteristic density distribution. Unconnected black dots in the highlights are nearly circular, as in (1), Fig. 7.

They are differentiated sharply into a central area of low density surrounded by a marginal ring of high density and relatively uniform width. The variation in width of the ring is of the order of 50 per cent. In the middle tones, the black dots print as variously curvilinear quadrangles, and the marginal ring is irregular in width, taking the general form shown in (2), Fig. 7. The center is denser than in (1), and the width of the ring may vary by several hundred per cent. In the narrow parts of the ring, its inner edge is sharply defined, but in the widest parts it is less sharp. In the shadow tones, the circular white dots are surrounded by a ring of high density, which is seen somewhat indistinctly against a dense background. The edge of the ring is less strongly defined than (1) or (2), and the variation in width is of the same order as in (1); see (3), Fig. 7. Excellent photomicrographs of printed dots have been published by A. J. Bull [13] and his associates.

The ratio of ring to dot might be accurately and quantitatively studied by means of enlargements from photomicrographs, but such an investigation would be outside the scope of the present work. While micrometering the black ink proof of Table II, Column 3, the width of the ring was measured at the widest and narrowest points for several dots in each tone, and a mean of the measurements taken as the average ring width. These values are listed in Table III, Column 1. The order of accuracy declines from ± .002 mm. in highlight steps Nos. 1, 2, and 3, to ± .005 mm. in the middle tones and shadows. The general similarity of the values in Columns 1 and 2 suggests that ring width and spreading may be closely connected, and the comparative regularity of numerical increment in the second column suggests that it may be easier to measure spreading in terms of ring width than by comparing dimensions of plate and proof. In Fig. 8, the ring widths from Columns 1 and 2, Table III, are plotted in mms. against original densities. When lines are drawn through each point, the curves obtained are seen to be closely related, the curve for Column 1 being the more erratic. The large increase in spreading for the first step of the latter is of little practical importance, as its effect on density is insignificant in the highlight region.

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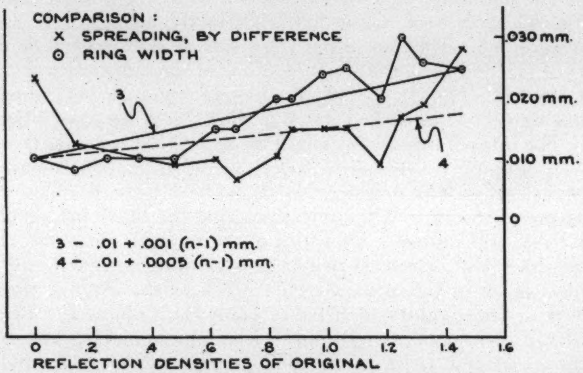
[13] A. J. Bull, "Photo-Engraving" (1934), Plates I and II, 150-line photo-engravings on B. P. chromo and a coated paper.

If these curves represent spreading with appreciable accuracy, it becomes apparent that any calculation of spreading must include an increment factor. The straight line (3), Fig. 8, agrees roughly with the ring width curve. It gives a spreading value of .01 + .001 (n − 1) mm., where n = the step number in a scale of 16 steps differentiated by equal density increments. The straight, dotted line (4), Fig. 8, is plotted for half the above increment or .01 + .0005 (n − 1) mm.

FIG. 8.

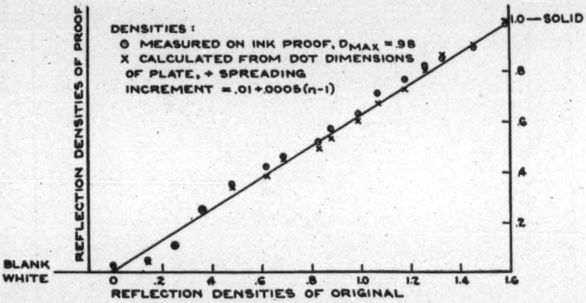


These spreading formulæ were tested experimentally. Starting with the plate measurements given in Column 1, Table I, the quantity .01 + .0005 (n − 1) mm. was added to the original black dot radii, or subtracted from the white dot radii, for each step, and densities were recalculated from the resulting dimensions for an ink of $D_{\text{max.}}$ = 1.0. These are plotted in Fig. 9 with the measured densities of a proof of $D_{\text{max.}}$ = 0.98 against original densities. Agreement is substantially perfect between proof and calculated curve.

Going back to the plate dimensions again, densities were recalculated for an ink spread of .01 + .001 (n − 1) mm. and ink density of 1.5. These were plotted in Fig. 10, together with densities from a proof in ink of $D_{\text{max.}} = 1.46$. Again, the agreement is almost perfect. Furthermore, the agreement is closer in Fig. 10 than in Fig. 6, in which densities calculated from micrometry of the proof are compared with densities measured directly from the proof.

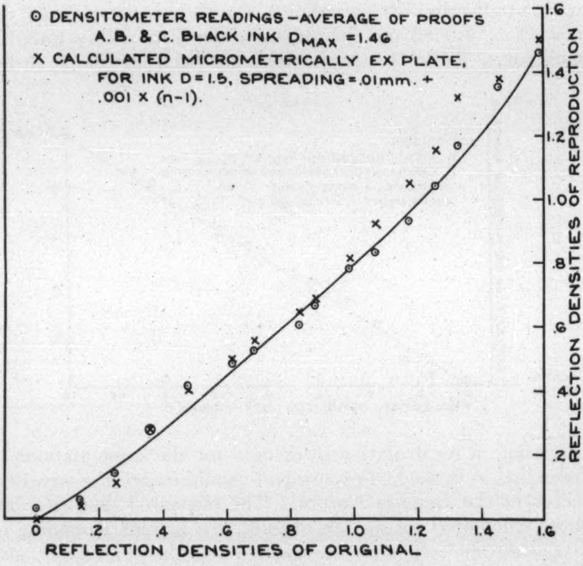
One of the proofs recorded in Column 3, Table II, was varnished to produce a high gloss and thus increase the density without affecting ink spreading. For the varnished proof, the new $D_{\text{max.}} = 1.88$ and the measured densities will be found in Column 5, Table II. Since there has been no change in ink spreading, a recalculation of density for the same amount of spreading as in the last experiment should match the measurements of the varnished proof. The measured densities and those calculated for an ink density of 1.9, plus spreading of $.01 + .001\ (n - 1)$ mm. are plotted together in Fig. 11 and agree almost perfectly.

FIG. 9.



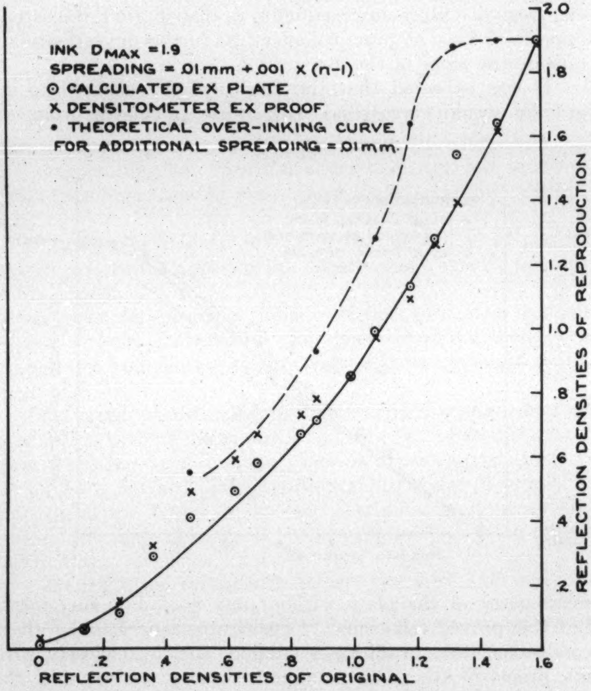
When the proof in which high density has been attained by varnishing is compared with curve "C," Fig. 5, in which high density was produced by extra heavy inking, it is seen that the former curve, Fig. 11, exhibits no "shouldering" in the shadows, while the latter had developed a marked shoulder. The pictorial result of shoulder is complete loss of differentiation in the deepest shadow tones, with a steep and sudden break from those tones to the lighter shadows. It had been observed that a shoulder could be created on an otherwise straight density curve by merely over-inking. How this occurs can be explained in terms of ink spreading. From the plate dimensions, an ink spread of .02 + .001 $(n - 1)$ mm. was calculated and the result converted into density for an ink of $D_{\text{max.}} = 1.9$. The densities were also plotted as a dotted line in Fig. 11. A great change has been brought about by an increase in spreading of only .01 mm. over the previous curve. A characteristic shoulder has appeared, and the shape of the curve closely resembles curve "C," Fig. 5, produced experimentally by over-inking.

FIG. 10.



It appears, then, that it would be difficult or impossible to secure straight-line reproduction of densities in photoengraving if it were not for ink spreading. Spreading can raise a definitely concave curve in the plate to a straight line in the proof. Increase in spreading beyond the optimum causes an undesirable shoulder in the shadow regions, and filling-in of the deeper shadows. Criteria of reproduction must take this phenomenon into consideration.

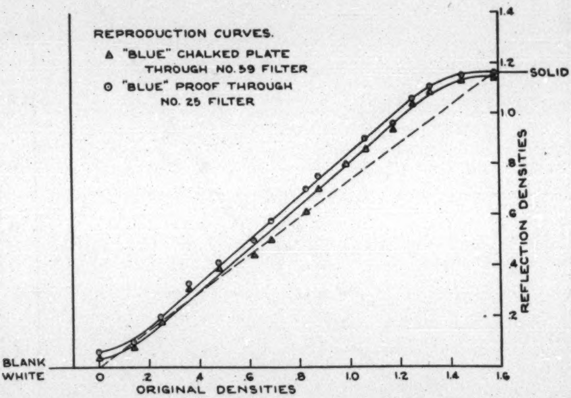
FIG. 11.



Because the introduction of a linear spreading increment produces curves by calculation that match measured densities without any allowance for ink distribution within the dot, it is tentatively assumed that the average ink distribution over the area of every dot on any one proof is identical. No direct measurements have been made, however, and, while the above is probably true for the shadow regions, the distribution could depart considerably in the highlights from the assumed condition without producing any notable effect upon density in that region. Since any inequality of distribution, if existent, appears to have no practical effect, its further investigation is outside the scope of this paper.

It was expected that the densities of a chalked-up engraving would correspond with those calculated from the micrometry of the plate without any spreading increment, but this proved fallacious. Densities measured under these conditions have, in all cases studied, coincided closely with ink proofs of the same maximum density, except that the chalked-up plate tends to give steeper gradation in the extreme shadows. This feature is variable, some plates giving a straight-line curve, almost identical with a low density ink curve of, say, $D_{\text{max.}} = 1.0$ to 1.2. See Fig. 12.

FIG. 12.



Evidently there is a contraction effect in chalking corresponding to the ink spreading effect in printing. The "chalking" operation consists in rubbing a block of magnesium carbonate over the surface of a plate until the depressions are filled with the white pigment. The excess is then removed by wiping with the side of the hand. This has the effect of removing some chalk from the depressions so that the chalk deposit is no longer flush with the printing plane. When measured in a densitometer with light incident at a 45° angle, a shadow is cast by one edge of the dot. This shadow is believed to have, fortuitously, an effect of the same order as ordinary ink spreading.

Through the foregoing considerations, a few facts may be tentatively accepted as reasonably certain.

Ink spreading has a greater effect upon the reproduction curve than variations in ink density likely to be met with in practice.

It cannot be said of any plate that it will yield perfect proofs except with a specific ink density and a specific amount of ink spread.

At least for normally inked 150-line proofs on enamel paper, without makeready, the dense marginal ring in a printed dot coincides approximately with the amount of ink spreading.

As a rough approximation, a plate giving straight-line reproduction with an ink of one $D_{\text{max.}}$ may be assumed to retain its straight-line form for changes in ink density up to ±0.3.

Given a finished plate, reproduction curves, of proofs on smooth paper, based on specific spreading increments and specific ink densities, can be accurately forecast from micrometric data.

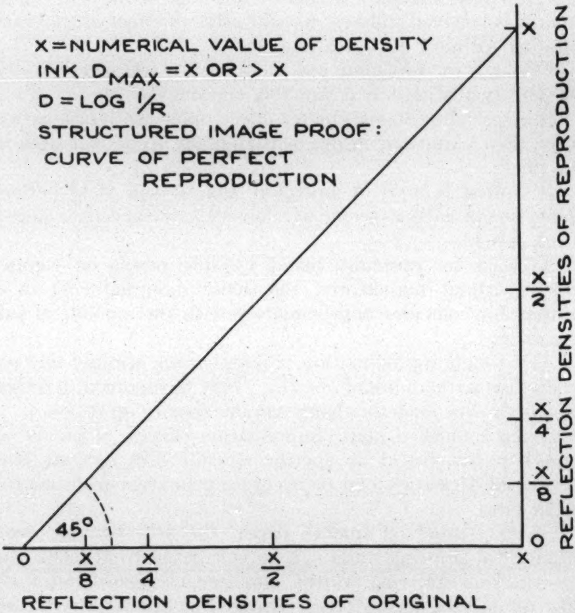
Given a proof on smooth paper, the area and per cent structure of printing units can be calculated from measured densities and ink ring widths, but linear dimensions of all dots can only be accurately determined by micrometry of the plate.

Given a plate and a proof in ink of one $D_{\text{max.}}$, reproduction curves can be calculated for other ink densities, provided spreading remains constant dimensionally.

In a process in which screen negative and etching conditions were held constant, linear dot dimensions in the plate might be calculated from densities and ring width on the proof by the use of predetermined data.

It is impractical to calculate ideal structure curves for any assumed spreading formula, for various ink densities, because the quantitative effect of spreading varies with the dot formation, which, for any given per cent. area, is at present unpredictable.

FIG. 13.



It is possible to specify under what conditions, if any, of ink density and spreading, a specific plate will yield a proof giving a perfect reproduction curve.

For control of daily production, micrometry is too laborious. As the object of the measurements is to secure data for the calculation of density, it seems more efficient to measure densities directly.

FIG. 14.

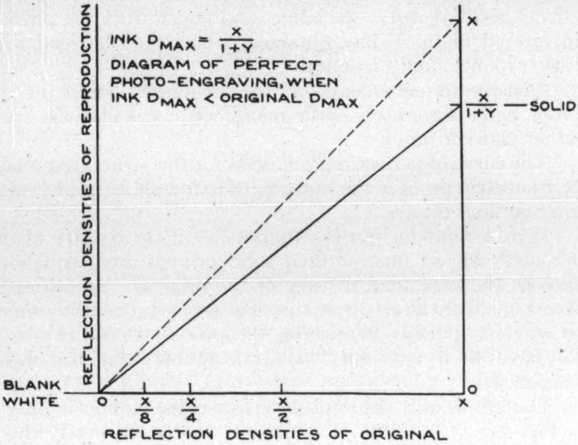
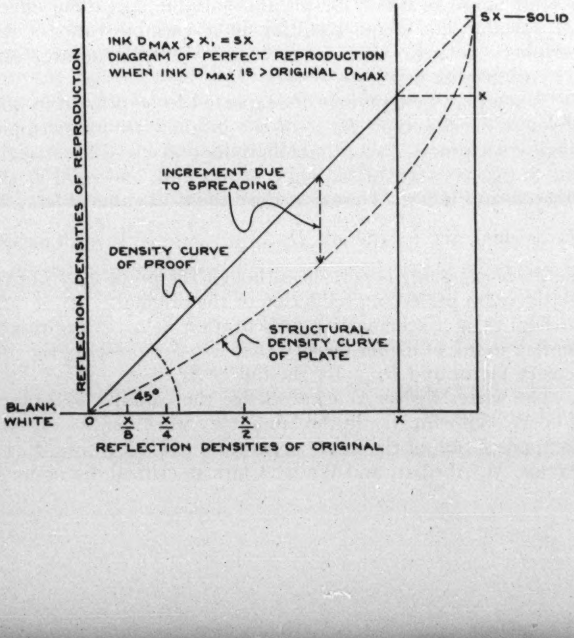

INK $D_{MAX} = \frac{X}{1+Y}$
DIAGRAM OF PERFECT
PHOTO-ENGRAVING, WHEN
INK D_{MAX} < ORIGINAL D_{MAX}
X
$\frac{X}{1+Y}$ —SOLID
REFLECTION DENSITIES OF REPRODUCTION
BLANK
WHITE
O
$\frac{X}{8}$
$\frac{X}{4}$
$\frac{X}{2}$
X
O
REFLECTION DENSITIES OF ORIGINAL

FIG. 15.


INK $D_{MAX} > X, = SX$
DIAGRAM OF PERFECT REPRODUCTION
WHEN INK D_{MAX} IS > ORIGINAL D_{MAX}
SX —SOLID
X
INCREMENT DUE
TO SPREADING
DENSITY CURVE
OF PROOF
STRUCTURAL
DENSITY CURVE
OF PLATE
45°
REFLECTION DENSITIES OF REPRODUCTION
BLANK
WHITE
O
$\frac{X}{8}$
$\frac{X}{4}$
$\frac{X}{2}$
X
REFLECTION DENSITIES OF ORIGINAL

When reflection densities are measured on the proof, the effects of ink density, spreading, and paper stock are already integrated in the values obtained, which may be read with ease, rapidity, and certainty.

Density measurements on the structured image of the proof agree accurately with micrometric calculations from either plate or proof.

The curve of perfect reproduction for the structured image of a halftone proof is the same as that for photography with unstructured images.

Perfect reproduction is only possible if the density of the ink used, for an unstructured solid printed area, equals or exceeds the maximum density of the original. Densities in excess of 1.6 are largely a matter of gloss, and generally cannot be reached without varnishing the proof. Densities above 2.0 have been read on black ink proofs that had been varnished.

The graph of a theoretical perfect reproduction is drawn in Fig. 13. The writer is at present unable to specify theoretical plate structures corresponding to such a proof. It is believed that they would have to be represented graphically by a solid figure to allow for (*a*) the variable spreading effect, (*b*) variable dot shape resulting in a variable ratio of dot periphery to area for any given per cent. structure, and (*c*) variable ink density.

When photo-engravings are expected to be printed in inks of lower density than $D_{\text{max.}}$ of the original as, for example, single color proofs, the optimal reproduction curve is a straight line drawn between blank white at one end, and solid at the other, as in Fig. 14. Here x = the numerical value of $D_{\text{max.}}$ in the original, and for the ink, $D_{\text{max.}} = \frac{x}{1 + y}$. In such a case, the *photo-engraving* is perfect, although the proof may not be considered a perfect reproduction of the original.

Fig. 15 is a schematic representation of a perfect reproduction in ink of higher density than x. S represents the ink density factor and $D_{\text{max.}}$ for the ink $= Sx$.

The writer wishes to acknowledge the valuable assistance of G. W. Wilhelm, Jr., in the laboratory work, and to express his appreciation of the kind coöperation of L. A. Jones, E. R. Davies, M. Abribat, and Walter Clark in critical discussions.

A STUDY OF THE QUALITY, THE DESIGN AND THE ECONOMY OF CONCRETE.

BY

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(*Continued from May issue.*)

PART II. DESIGN OF CONCRETE MIXES.

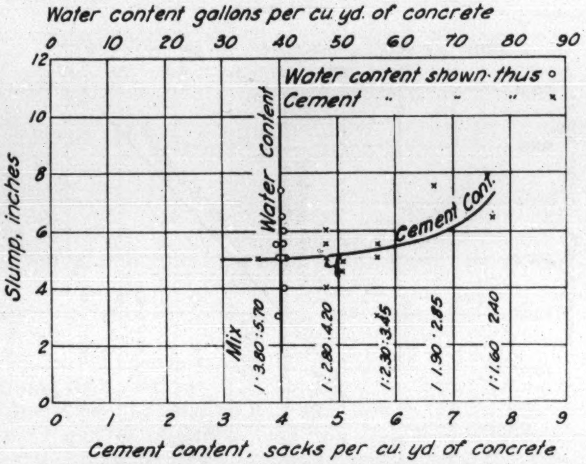
All design of concrete mixes is made for the purpose of determining the combination of ingredients which under given conditions, will produce the desired qualities of the concrete. Since a great number of different mixes will give the desired results, the important item is to find that combination which will produce the given qualities at the least cost. The cement is the most expensive ingredient in the concrete and the cementing paste is the quality-giving portion of the mix, so the problem of design often becomes that of finding the combination of the fine and coarse aggregates which will make the concrete placeable with a minimum amount of cement. However, the unit prices of the materials determine to some extent, the type and grading of the aggregates because the cost of the hardened concrete is the criterion for economy. Often low grade aggregates incorporated in a high grade paste will produce concrete of the desired strength and durability at less cost than will high grade aggregates. Since the object is to secure a final product of a given quality, the attention of the engineer should be focused principally upon the quality of the hardened concrete and not be diverted to consideration of quality and standards in the ingredients used, beyond the point of useful returns. The method to use for obtaining the most economical quality of the concrete is studied more fully in Part III of this paper.

The trial method of designing concrete mixes has gradually replaced most of the others, including the fineness modulus and the surface area methods. By the trial method the most economical mix for a given quality of concrete is determined from successful trials in which the materials required for a unit volume of concrete are measured for different combinations of the available supplies. The whole purpose of these trial mixes is to find that combination of materials which, for a given method of placing, will produce the most economical concrete of a given quality. The extent to which refinements should be carried in this search with trial batches for the most economical product, is to a large degree determined by the size of the concrete project.

a. Constant Water Content.

During the making of a large number of different concrete mixes at the laboratory of the Portland Cement Association, the author noted that the amount of mixing water per unit of concrete remained much the same, regardless of the richness of the mix, so long as the consistency remained nearly the same. He decided to make use of this fact in the design of the beam investigation (23) which was undertaken immediately after he came to Lehigh University. The results were so encouraging that the so-called constant water content method was adopted for the design of all future investigations, including the very extensive one on reinforced concrete columns. The experiments with the ordinary sand and gravel aggregates used in the laboratory, indicated that a net water content of approximately 40 gal. per cu. yd. of concrete would be necessary for the desired placeability when the sand-coarse ratio was 1 to 1½ by weight. The net water content was therefore maintained at about 40 gal. per cu. yd. for all the mixes included in the investigation. The cement content ranged from as low as 3.6 sacks per cu. yd. for the lean mix to as much as 7.7 sacks for the very rich mix. The interrelation between consistency of concrete as measured by the slump, the cement content in the mix, and the total water content used are shown in Fig. 22. It is noted that the average results for the slump do not vary much for the different cement contents. The slump for the concrete containing 3.6 sacks cement is just about the same as that for the concrete containing 7.7 sacks cement per cubic yard of concrete. Since the net water content remained constant for these mixes, the cement-water ratio of the paste would naturally be directly proportional to the cement content, and the strength should increase in proportion to the increase in cement content in the

FIG. 22.

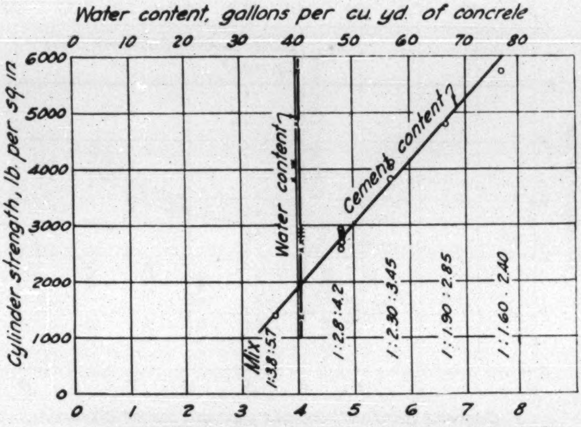


Interrelation between water content, cement content and slump of concrete.

concrete. The results showed this to be true as is illustrated in Fig. 23, in which the relationship between strength, cement content, and total water content is given. This figure proves conclusively that the important factor for the quality of the concrete is the cement content, and not the water content which was so much advocated when the water-cement ratio was first brought forth. It must be kept in mind that the gradation of the aggregates as determined by the fine-coarse ratio remained constant for all these mixes. If the fine-coarse ratio had been altered, the water content for a given consistency would not remain constant. A mix having a larger amount of fines in its aggregate than another mix will naturally require more water for the same consistency, provided both mixes are placeable.

In order to study the effect of the type of cement and type of aggregates on the water content of the mix, two series of tests were made (24). The first series had siliceous sand and gravel as aggregate for all the concrete mixes. The sand had a fineness modulus of about 2.80 and was well suited for concrete. The gravel was graded so as to contain 40 per cent. by weight between No. 4 and ⅜-inch sieves and 60 per cent. between ⅜ and ¾-inch sieves. For one group the sand-coarse ratio was 1 to 2 by volume of dry rodded materials or 1 to 1.99 by weight. Five entirely different types of cements were used. The net water content was 36 gallons, or 300 lb. per cu. yd. of concrete for all the mixes, regardless of the type of cement. The results of the slump observations are given in Table I, in which the averages for three batches of concrete are given. It is noted that the slump for each type of cement remained nearly the same for all four mixes used. The average slump for each cement is seen to vary from as low as 1½ inches for

FIG. 23.



Effect of cement content and water content on strength of concrete.

TABLE I.

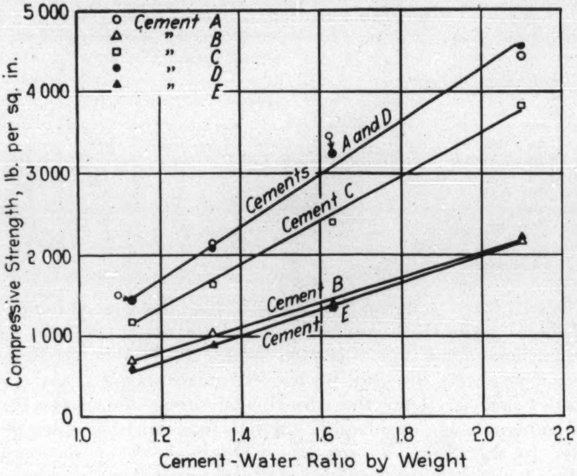
Slump of Concrete Mixes Containing Five Different Brands of Cement.

Mix by Weight of Dry Materials.	Slump in Inches.					
	Cement *A.*	Cement *B.*	Cement *C.*	Cement *D.*	Cement *E.*	Average.
1 : 1.64 : 3.26.....	2½	1½	3	2½	5½	3
1 : 2.19 : 4.35.....	2¼	1½	5¾ *	2½	4½	3¼
1 : 2.74 : 5.43.....	3¾	1½	5	2	4½	3½
1 : 3.29 : 6.51.....	3	1½	2½	3	5	3
Average......	3	1½	4	2½	5	3¼

* Fell down.

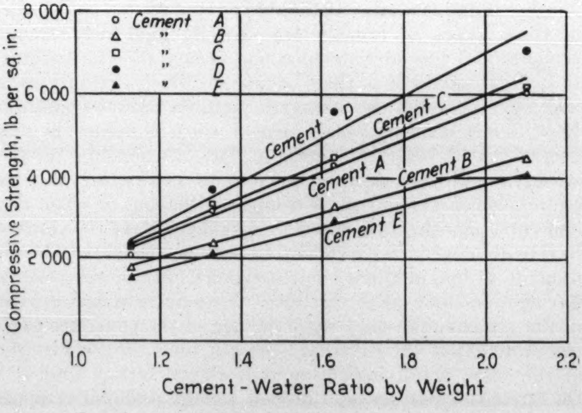
Cement *B* to 5 inches for Cement *E*. The last column of Table I gives the average slump for the same concrete mix regardless of the type of cement used. These average slumps are very nearly the same for the four mixes, being 3, 3¼, 3½, and 3 inches. Thus the variation in slump due to the type of cement was considerably greater than was the variation due to the change in the mix. However, the variation in slump due to the type of cement was so small that for ordinary cases it may be assumed that a mix which is placeable when one brand of cement is used will also be placeable when another brand is used. It should be remembered that certain of these types of cement were special high-early-strength cements and special water-proofing cements. The variation in strength qualities of these cements is illustrated in Figs. 24 and 25. Although the strengths for the concrete containing the different cements varied greatly, they all follow the same general trend, that is a straight line relationship between strength and cement-water ratio. In comparing the two figures, which give strength results at the ages of seven days and three months, it is noted that at seven days the concrete having Cement *A* is considerably stronger than that having Cement *C*, but at three months the strength of the concrete having Cement *C* is the greater. Thus the strength quality of the cement may vary with the age of the concrete. It is also noted that the different strength lines do not intersect at the same point, indicating that the factors *A* and *B* in the strength equation are different for the different cements.

FIG. 24.



Strength-cement-water-ratio relationship at the age of seven days.

FIG. 25.



Strength-cement-water-ratio relationship at the age of three months.

Another group of the first series had the same kind of aggregate as the first group, but the sand-coarse ratio was 1 : 1½ by weight. Thirteen different brands of standard portland cement and five brands of special cements were included. The net water content was 39 gal. per cu. yd. of concrete for all four mixes used, and for all the cements. The four cement-water ratios were 1.25, 1.58, 1.92 and 2.25 by weight. The results of the observed slumps are given in Table II for each brand of cement and each cement-water ratio. The average for each cement and the average for each cement-water ratio are given for the standard portland cements and for the special cements. There is a remarkable uniformity in the observed slumps, both for the four mixes and for the eighteen cements. The average results varying only between 2½ and 4¾ for all the cements used. These tests indicate very strongly that for the ordinary range of concrete mixes as well as for the ordinary types of cements, the constant water content theory forms a sufficiently accurate basis for the design.

In the second series of tests, nine widely different types of aggregates were used with the same brand of cement. Three concrete mixes were made with each type of aggregate and the fine-coarse ratio was 1 to 1½ by weight for all mixes. The results of the consistency tests are shown in Table III which contains a statement of the type of aggregates, the nominal net water content for each aggregate and the observed slumps for the three mixes. In certain cases slight additions to the nominal water contents were made for some of the mixes as indicated in the table. It is noted that for approximately the same slump, the nominal water content varied from as low as 37 gallons to as much as 45.3 gallons per cubic yard concrete. However, for each type of aggregate the slump did not differ much for the three mixes having the same water content. Since the cement-water ratio by weight varied from as low as 1.25 to as high as 2.25, it may be concluded that for ordinary conditions the consistency of the concrete, as measured by the slump, remains nearly constant, regardless of the richness of the mix, as long as the type and gradation of aggregates and the water content per cubic yard of concrete remain constant. This uniformity in the consistency of

TABLE II.

Slump of Concrete Having Constant Water Content.

Cement No.	Slump in Inches for Cement-Water Ratio, by Weight.				
	1.25.	1.58.	1.92.	2.25.	Average.
1	3	3½	6 *	4	4
2	2½	1½	2½	3	2½
3	3½	2	2½	3	2¾
4	2½	5	6 *	6 *	4¾
5	2	3	3½	3½	3
6	3	2	4	3	3
7	2½	2½	4	4½	3½
8	3½	3	3	6 *	3¾
9	3	3	3	3½	3
10	3	3½	3	3½	3¼
11	2½	2	3	3	2½
12	3	3½	4½	3	3½
13	4	4	3	4	3¾
Av. 1–13	3	3	3¾	3¾	3½
A	3	3½	2½	3	3
B	3	2½	2½	3	2¾
C	3	2	2½	2	2½
D	5	2	3	3	3¼
E	3	4	3½	3	3½
Av. *A*, *B*, *C*, *D*, *E*	3½	2¾	2¾	2¾	3

* Fell down, included in average.

concrete of a given water content is very useful in designing concrete mixes. Since the water content remains constant, the density of the concrete also remains constant, that is, the amount of solids per unit of concrete remains constant. Thus a change in the cement content made necessary for a change in the quality of the concrete, requires a corresponding change in the aggregate content if the consistency is to remain the same. These changes mean changes in the absolute volumes of the ingredients in the unit volume of concrete. The fact that the water content is nearly constant for concrete mixes of different richnesses is made use of in the simplified trial method for designing concrete mixes of given consistency using a given type and gradation of aggregate (25). Since any method of design is only a convenient tool

TABLE III.

Slump of Concrete Mixes Containing Nine Different Aggregates.

Aggregate No.	Type of Aggregate.		Net Water Content Gal. per Cu. Yd.	Slump in Inches for Mix Having Cement Water Ratio of:			Average.
	Fine.	Coarse.		2.25.	1.61.	1.25.	
1	Limestone Screenings	Crushed Limestone	45.3	2	2¾ [a]	2¾ [a]	2½
2	Sand	Gravel	37.0	2	3 [b]	3 [b]	2¾
3	Sand	Pea Gravel	44.1	3½	3½	4	3¾
4	Sand	Gravel	38.0	1¾	1	1½	1½
5	Limestone Screenings	Crushed Limestone	45.3	5	5	6	5¼
6	Limestone Screenings	Crushed Limestone	43.0	½	1	1	¾
7	Sand	Gravel	40.5	½	2¼ [c]	2¼ [c]	1¾
8	Sand	Pea Gravel	41.5	4¾	5	3½	4½
9	Sand	Gravel Partly Crushed	41.5	3½	5	3	3¾
			Average	2.6	3.2	3.0	2.9

[a] 0.7 gal. per cu. yd. added to mix.
[b] 0.5 gal. per cu. yd. added to mix.
[c] 1.0 gal. per cu. yd. added to mix.

for the determination of the nearly correct quantities of materials per batch of concrete, with necessary small final adjustments to be made directly at the mixing plant, the principle made use of in this simplified method is sufficient accurate for any ordinary condition, and it has its advantage over all other methods in the resulting fewer trial batches.

b. Simplified Method of Design.

The first step in this method for designing a concrete mix is to determine by trial batches that gradation of the available aggregate which is most economical for the production of a concrete of a given quality. The next step is to determine also by trial batch, the amount of water necessary to produce the desired consistency of the concrete which has the already fixed gradation. One trial batch is sufficient for the determination of the amount of mixing water. The remainder of the design is a matter of computation. We have (assuming no air voids which is very nearly true for properly placed concrete) that:

$$V = V_w + V_c + V_a, \tag{10}$$

where V = volume of concrete,
V_w = volume of mixing water,
V_c = absolute volume of cement,
V_a = absolute volume of aggregate.

Expressing the amounts of ingredients in terms of weight we have:

$$V = \frac{w}{g_w} + \frac{c}{g_c} + \frac{a}{g_a}, \tag{11}$$

where w, g_w, c, g_c, a, and g_a represent the weights and specific gravities of water, cement and aggregate respectively. If D is the density, or the ratio of solids, in a unit volume of concrete, then:

$$D = \frac{c}{g_c} + \frac{a}{g_a} \tag{12}$$

For a given water content the density of the concrete is constant so that a change in the cement content must equal a change of the same magnitude in the absolute volume of the aggregate, or:

$$\frac{c'}{g_c} = \frac{a'}{g_a}, \tag{13}$$

where c' and a' are the changes in weights of cement and aggregate, or:

$$a' = c' \frac{g_a}{g_c}, \tag{14}$$

that is, a change of one pound in the cement content will require a change of g_a/g_c pound in the aggregate content if the water content per unit of concrete shall remain the same. Thus in changing from a mix of one cement-water ratio to one of another ratio, all that is necessary is to multiply the change in cement-water ratio by the amount of water in the mix and make a change of g_a/g_c times the change in cement content in opposite direction, that is, a decrease in cement content requires an increase in aggregate content and vice versa. For ordinary aggregates and cements, the specific gravities are approximately 2.65 and 3.10 respectively. Consequently $g_a/g_c = 0.85$ for ordinary concrete mixes, or one pound change in the cement content requires 0.85 pound change in the aggregate content.

By this method only one trial mix is needed for all concrete mixes having a given type and gradation of aggregate designed for a given consistency, while the ordinary trial batch method would require at least one trial batch for each mix, or at least as many trials as number of mixes to be used.

c. Sample of Design.

Assuming that the trial batches on the type and gradation of aggregate resulted in the 1 to 2 weight ratio between fine and coarse as the most economical aggregate combination for the desired consistency of the concrete, the trial procedure would be as follows: Weigh out cement, fine and coarse aggregate in the ratio 1 : 2 : 4, or any other convenient ratio where the sand-coarse ratio is 1 to 2, mix these ingredients together with the addition first of the water necessary for the absorption of the aggregate, and then of known quantities of mixing water until the desired placeability of the concrete is obtained. Determine the volume of the batch by adding the absolute volumes of cement, aggregates, and net mixing water, and thus find the quantity of each ingredient necessary to produce one cubic yard of concrete. Assuming that the net water requirement was 300 lb. per cu. yd. of concrete, all the mixes would be designed on the basis of this water content. Practically all concrete mixes in present-day construction utilize cement-water ratios between 1.25 and 2.25 by weight. If these values are selected for the determination of the strength-cement-water-ratio relation for the materials used, we have for $c/w = 1.25$ a cement content of $1.25 \times 300 = 375$ lb. per cu. yd. concrete. Since the water occupies $300/(27 \times 62.4) = 0.178$ cu. yd. and the cement $375/(27 \times 62.4 \times 3.10) = 0.072$ cu. yd. in a cubic yard of concrete, the aggregate content is $(1 - 0.178 - 0.072) \times 27 \times 62.4 \times 2.65$ lb. $= 0.75 \times 4460$ lb. $= 3450$ lb. per cu. yd. of concrete. If the aggregate absorbed one per cent. water by weight, the absorption water will be 34.5 lb. so that the total water in a cubic yard of concrete is 334.5 lb. For the cement-water ratio of 2.25 the cement content is 2.25 × 300 lb. = 675 lb. or an increase of 300 lb. per cu. yd. of concrete. The decrease in the aggregate content is 0.85 × 300 lb. = 255 lb. so that the aggregate content for this mix is 3450 lb. − 225 lb. = 3195 lb. or 1065 lb. sand and 2130 lb. coarse aggregate per cubic yard of concrete. The absorbed water is 32 lb. per cu. yd. concrete.

If the compressive strength of the two mixes are 1600 and 4000 lb. per sq. in. for the 1.25 and 2.25 mixes respectively, the strength equation is:

$$f_c' = -1400 + 2400\frac{c}{w}$$

or

$$f_c' = -1400 + 8 \cdot c$$

where c is given in pounds per cubic yard of concrete. The cement content for concrete of any given strength is then determined:

$$c = \frac{f_c' + 1400}{8}$$

where c is in pounds per cubic yard of concrete. The whole procedure after the one trial batch has been made, is simply a few slide-rule computations for any mix which has the same gradation of aggregate and placeability of the concrete as did the original mix.

PART III. RELATION BETWEEN QUALITY AND ECONOMY OF CONCRETE.

The effect of the quality of the concrete on the economy of the plain or reinforced concrete members has long been a moot question. Different investigators have pointed out that a rich, and consequently high quality concrete, will in most cases be more economical than a lean and weak concrete, but no direct rational study has been available due to the large number of variables involved in the problem. Professor A. N. Talbot (26) during his early experimental work on concrete, pointed out that the richer concrete mix was more economical for producing a column of a given strength than was a leaner mix. Peter Gillespie (27) and M. O. Withey (28) also concluded from experiments on reinforced concrete columns, that a richer concrete was more economical than a leaner one and that increased richness of the concrete added strength more economically than did increased longitudinal reinforcement. Arthur R. Lord (29) recently presented data showing that a considerable saving was accomplished when the strength of the concrete was 3000 instead of 2000 lb. per sq. in. H. Olsen (30) concluded from an elaborate study of the economical advantages of high working stresses that an important saving was accomplished by increasing the working stresses to about 1400 lb. per sq. in. for the concrete and about 28,000 lb. per sq. in. for the steel, which would necessitate high strength concrete and high yield-point steel. An increase beyond these limits was found inadvisable on account of the relatively small increase in economy and the practical difficulty of producing materials of the necessary high quality.

However, no rational study had previously been given to the relation between the quality and economy of concrete members. The establishing of the nearly constant water content in concrete mixes of a given type and gradation of aggregates made it possible to make the rational study presented in this paper (31). Since all other desirable qualities of concrete except volume changes increase with the increase in the compressive strength, the strength relationship will be used as the criterion for the quality of the concrete.

a. The Economy of Plain Concrete.

1. *General.*

The total cost of concrete in place is composed of the cost of the materials plus the cost of formwork, mixing, placing, curing, finishing, etc. Since the cost of formwork, mixing, placing, etc. does not vary greatly with the concrete mixes of different richnesses as long as their consistencies are nearly the same, the variation in cost of the concrete is principally due to the variation in cost of the materials, that is, the cement and the aggregate. The cost of the materials in one cubic yard of concrete may be expressed by:

$$P = p_a \cdot a + p_c \cdot c \tag{15}$$

where p_a and p_c represent the unit prices, and a and c are the amounts of aggregate and cement in a cubic yard of concrete. It has already been pointed out that the total volume of properly placed concrete is the sum of the absolute volumes of aggregate, cement and water, or:

$$V = V_a + V_c + V_w.$$

Since the amounts of the individual ingredients are given in weight, we have:

$$V = \frac{a}{g_a} + \frac{c}{g_c} + w,$$

which gives

$$V = D + w,$$

where

$$D = \frac{a}{g_a} + \frac{c}{g_c}$$

or

$$a = D \cdot g_a - c \cdot \frac{g_a}{g_c},$$

which for ordinary materials gives:

$$a = D \cdot g_a - 0.85c. \qquad (16)$$

Substituting this value for a in the cost equation gives:

$$P = p_a(D \cdot g_a - 0.85.c) + p_c \cdot c \qquad (17)$$

or

$$P = D \cdot g_a \cdot p_a + c(p_c - 0.85p_a). \qquad (18)$$

The value g_a for one cubic yard of concrete is:

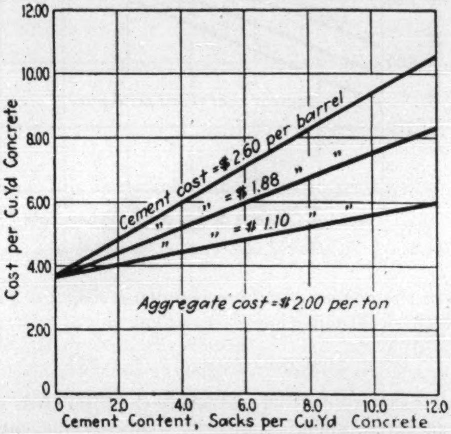
$$g_a = 2.65 \times 62.4 \times 27 = 4460,$$

so that the final cost equation is:

$$P = 4460 \cdot D \cdot p_a + c(p_c - 0.85p_a). \qquad (19)$$

Thus the cost of the materials per cubic yard of concrete varies directly with the cement content for given unit prices of aggregate and cement and for a given density of the concrete. In Figure 26 the relation between cost of materials and cement content of concrete is given for a price of $2 per ton for the aggregate, and prices of $1.10, $1.88, and $2.60 per barrel of cement. It is noted that the increase in cost with increase in richness of the concrete is very large, particularly for the high price of the cement, the cost increasing from about $5 for a cement content of 2 sacks per cu. yd., to about $10 for 11 sacks cement per cu. yd. of concrete. It is also noted that the price of cement has a marked effect upon the cost of the concrete, particularly for high cement contents.

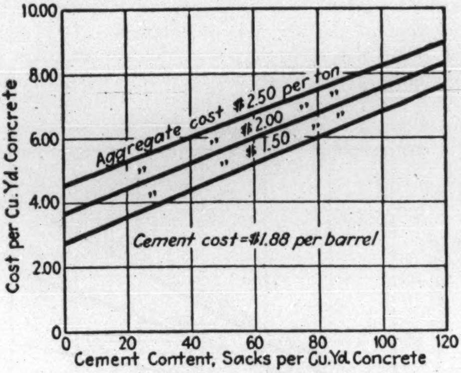
FIG. 26.



Effect of cement content on cost of concrete materials for different prices of cement.

Figure 27 shows how the price of the aggregate affects the cost of the concrete containing different amounts of cement. The price of the cement was taken as $1.88 per barrel, or ½ cent per pound, and the price of the aggregate was $1.50, $2.00 and $2.50 per ton. Since the aggregate content is only slightly affected by the cement content in the concrete, the variation in the price of aggregate is not much different for lean and rich mixes, being slightly greater for a lean than for a rich mix. The above studies were made on a concrete which required a net water content of 303 lb. per cu. yd. concrete for the desired placeability. The volume of water in the mixes was then: $303/(27 \times 62.4) = 0.18$ or 18 per cent. of the volume of the concrete making the density 0.82. This gives the following cost equation:

FIG. 27.



Effect of richness of mix on cost of concrete materials for different prices of aggregate.

$$P = 4460 \cdot 0.82 \cdot p_a + c(p_c - 0.85p_a). \tag{20}$$

In plain concrete structures where the total cross-sectional area is in direct compression, the load-carrying capacity is directly proportional to the strength of the concrete used. The cross-sectional area needed to resist a given load is thus:

$$A_c = \frac{F}{s \cdot f_c'}, \tag{21}$$

where A_c is area of concrete, F is total load, s is factor of safety, and f_c' is strength of concrete. Thus the cross-sectional area is inversely proportional to the strength of the concrete. Since the volume of concrete in a member of a given length is proportional to its cross-sectional area, this area represents the cost ratio for different concrete mixes.

The cost of the concrete materials per unit of strength is therefore a direct measure of the economy of the mix. Thus we have:

$$E = \frac{P}{f_c'} = \frac{1}{f_c'}(4460D\cdot p_a + c(p_c - 0.85p_a)), \qquad (22)$$

where E is the economy of the concrete in terms of cost per cubic yard per unit of strength. In order to express the economy of the mix in terms of strength of concrete instead of cement content, we make use of the previously presented strength equation: $f' = A + B\cdot\frac{c}{w}$ or for constant water content, $f_c' = A + \frac{B}{w}\cdot c = A + Kc$, from which:

$$c = \frac{f_c' - A}{K}. \qquad (23)$$

Substituting this value for c in the economy equation gives:

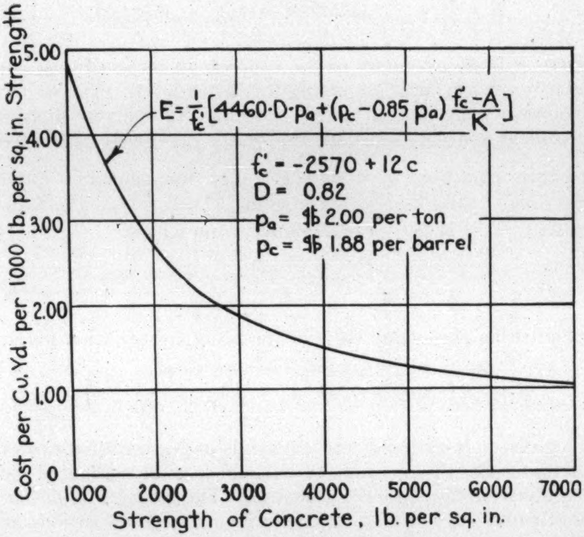
$$E = \frac{1}{f_c'}\left(4460D\cdot p_a + (p_c - 0.85p_a)\frac{f_c' - A}{K}\right). \qquad (24)$$

For given unit prices of the materials and given qualities of the materials, the economy of the mix may be computed for any given strength of the concrete. The strength equation for the materials recently used for concrete investigations at Lehigh University is:

$$f_c' = -2570 + 12c, \qquad (25)$$

where c is expressed in pounds per cubic yard of concrete, Using this strength equation for unit prices of \$2 per ton aggregate, and \$1.88 per barrel cement or ½ cent per pound. and a density of the concrete of 0.82, corresponding to a net water content of 303 lb. per cu. yd., we obtain the economy curve presented in Fig. 28. It is noted that when the economy of the materials in the mix is expressed in terms of cost per cubic yard of concrete per 1000 lb. per sq. in. compressive strength, a very marked increase in economy follows an increase in strength, particularly for relatively weak concrete. As the strength of the concrete increases, the gain in economy tapers off so that for very rich mixes the gain is relatively small.

FIG. 28.



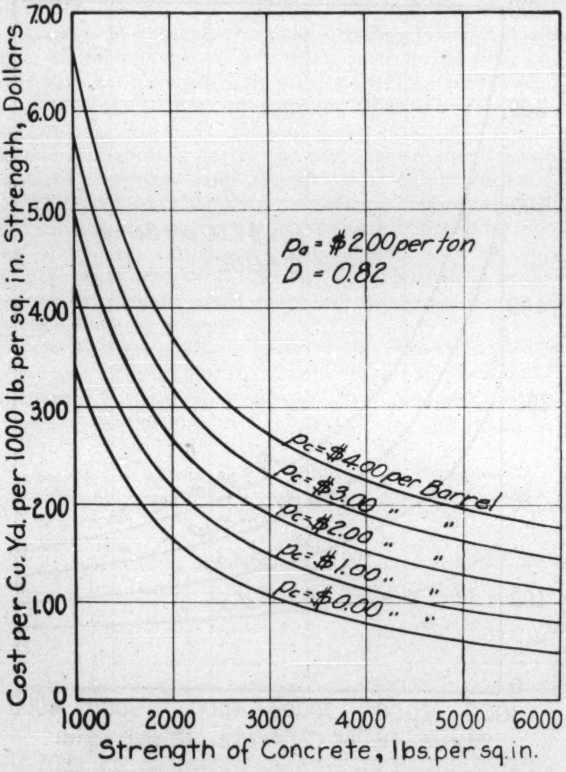
Effect of strength of concrete on economy of mix.

The above study is a valuable aid in the determination of the most economical mix and the selection of the proper types of cement and aggregate. In the following section a detailed study will be given to these problems.

2. *Effect of Unit Prices and Characteristics of Materials.*

In order to study the effect of the unit prices of cement and aggregate, Figs. 29 and 30 were prepared (32). The strength equation used for this study was: $f_c' = -2570 + 12c$, with a density of the concrete of 0.82 corresponding to a net water content of 303 lb. per cu. yd. of concrete.

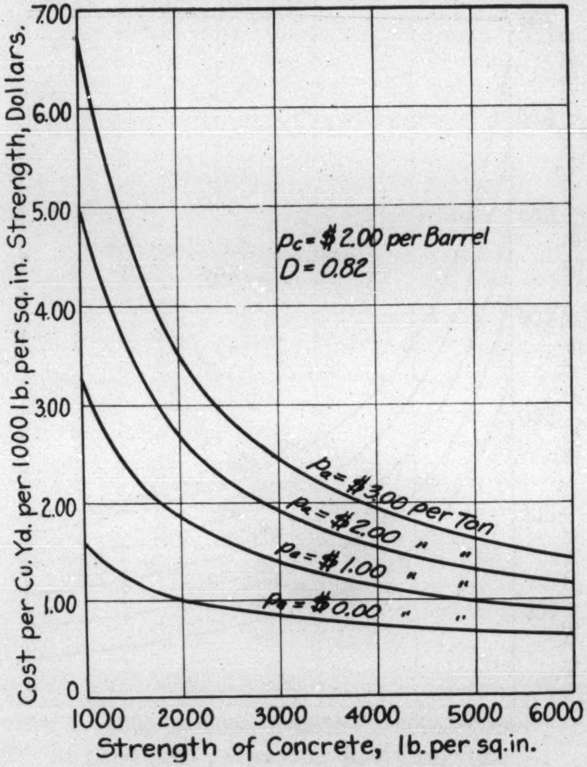
FIG. 29.



Relation between strength and economy of concrete for different prices of cement.

Figure 29 shows the relation between the economy of the materials in the mix and the strength of the concrete for five different unit prices of the cement, that is, 0, $1, $2, $3, and $4 per barrel. The price of the aggregate was taken as $2 per ton. A very great decrease in cost with the increase in

FIG. 30.



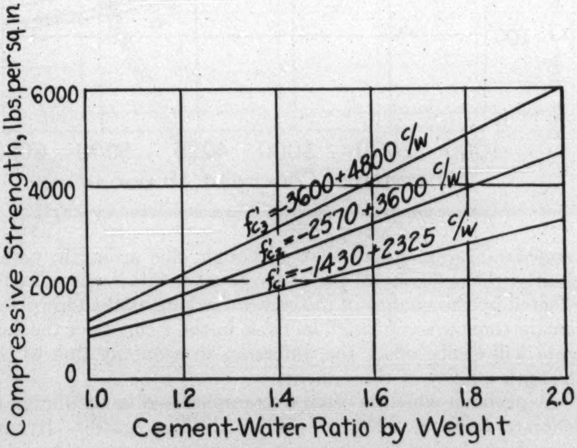
Relation between strength and economy of concrete for different prices of aggregate.

strength of the concrete is noted. As a matter of fact, a change in the cement content, which will produce a corresponding chance in the strength, will easily offset small differences in the price of the cement. The difference in economy due to $1 per barrel in the cement price is nearly $1 for 1000 lb. per sq. in. strength of the concrete. For concrete of a strength of 6000 lb. per sq. in. this difference is only about $0.35.

In Fig. 30 the only variable introduced was that of the price of the aggregate. For weak concrete the price of the aggregate has a considerably greater effect upon the economy of the concrete than a corresponding difference in the price of the cement. However, for strong concrete the difference in economy is affected nearly the same by an equal difference in prices of aggregate and cement. While a difference of $1 in the price per ton of aggregate makes a difference of about $1.75 in the economy of a concrete with strength of 1000 lb. per sq. in., the difference is only about $0.30 for concrete with strength of 6000 lb. per sq. in. Thus the price of the aggregate becomes a much more important item for lean than for rich concrete mixes.

The strength-giving quality of the cement is always an important item on any construction job, and the economical advantage of this quality may well be studied by means of the formulas developed. In Fig. 31 the strength relations for

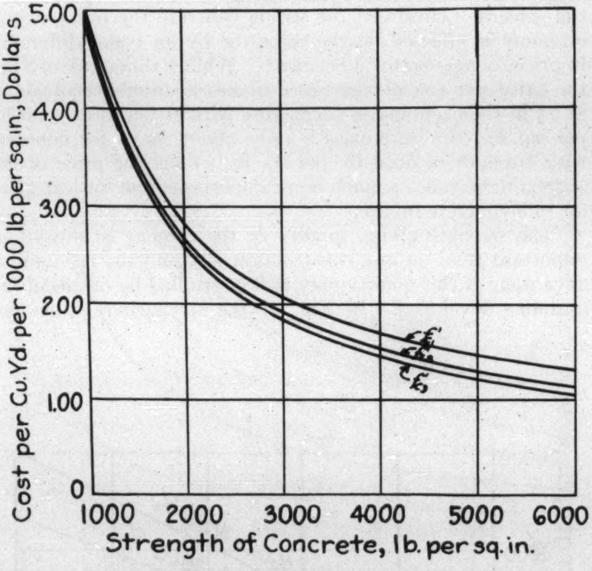
FIG. 31.



Strength relations for three different cements.

three different grades of cement are plotted. There is a great difference in the strength obtained with the three different cements for any given cement-water ratio. Fig. 32 shows the economical relationship for the concrete containing these cements. It is noted that although the strength varied greatly, the economy of the concrete materials is only slightly affected by the quality of the cement as long as the unit prices remain constant. A small increase in the richness of the concrete will easily offset the difference in economy due to the strength quality of the cement.

FIG. 32.



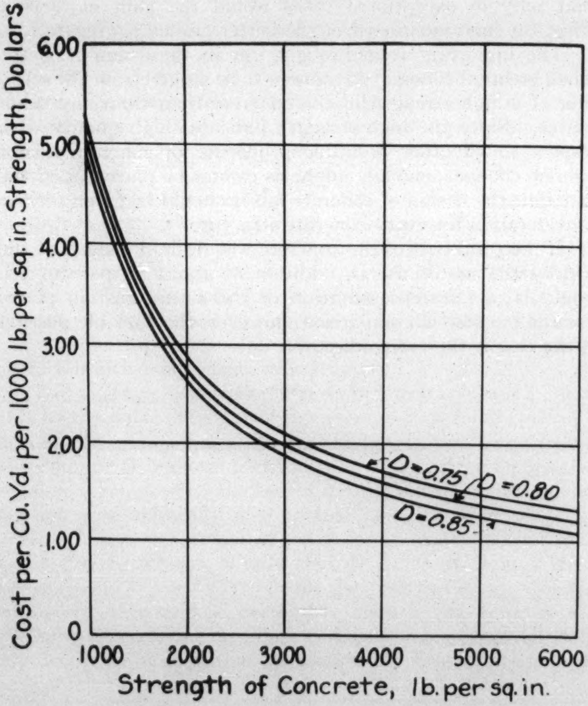
Effect of strength-giving quality of cement on economy of concrete.

A problem which is often overemphasized in the design of concrete mixes is the gradation of the aggregates. By improved gradation the water requirement is reduced and a corresponding reduction is made in the cement content in order to maintain the cement-water ratio the same. Practical experiences have shown that a high degree of artificial grading of the aggregate is not economically justified for ordinary concrete construction, but much doubt has existed as to how much attention should be given to the gradation problem. In order to study this problem the concrete of the strength-quality:

$$f_c' = -2570 + 12c$$

was used as the basis for the analysis presented in Fig. 33.

FIG. 33.



Effect of gradation of aggregate on economy of concrete.

In preparing this figure, it was assumed that the strength of the concrete followed the above equation regardless of the gradation of the aggregates and the consequent variation in the density of the concrete. The variation in gradation was carried as far as to change the density of the concrete from as low as 0.75 to as high as 0.85, or a change in the net water requirement from as high as 421 lb. to as low as 253 lb. per cu. yd. of concrete, which is a much greater change than for ordinary cases. The economy of the concrete having these, gradations varied only slightly when the cost was $2 per ton for the aggregate and $2 per barrel for the cement, indicating that only in exceptional cases would the gain in density offset the increase in cost of the better graded aggregate.

The important item brought out in these figures is the much greater economic advantage to be gained from the selection of a high strength of the concrete than from any other source. Since the high strength indicates high quality with respect to all other desirable properties of concrete except volume changes, enough emphasis cannot be placed upon the fact that the design of concrete mixes should be given serious consideration for every concrete structure.

If the total cost of the concrete was desired instead of the cost of the materials, an addition of about $4 per cu. yd. would be a fair representation of the actual cost in place. For the purpose of comparison, however, the variable portion of the cost is the more adaptable one.

(*To be continued.*)

RANDOM FRACTURE OF A BRITTLE SOLID.

BY

C. C. LIENAU,

University of Chicago.

(*Concluded.*)

PROPAGATION AND FORM OF NEW SURFACE; FRAGMENT SHAPE PROBABILITIES.

From the scale of observation Δ^3, a fissure does not definitely exist until its surface has become Δ^2. A homogeneous strain in a uniform material would predispose the fissure form toward the sphere. But statistical fluctuations of structure and stress tend in a real material to become appreciable at the level Δ so that, in general, the fissure will exhibit a non-spherical form. The physical instant of time may be defined as the time of formation of the complete fissure. This interval is short in comparison with τ but we examine its fine content in a manner analogous to that for the physical point, the ultimate event being here the formation of the elementary fissure. This takes place in an interval which we cannot or do not choose further to analyze. The ultimate analytical element in time is the fully formed spheroidal void whose surface is either increasing or stationary.

The average displacement of complete fracture is l. Let this be the polar diameter of the average completed spheroid, and let the orientation of the pole be $\boldsymbol{n}$. In the average solid the equatorial plane is then tangent to a surface of positive tension as required by the assumption of tensile failure. This tension was uniformly $u \pm \delta p$ just prior to fracture. The tension is now tangential to the spheroid at all points. The polar surface tension is only slightly different from u since the spheroid is very flat. From the pole to the equator the radius of the surface decreases. Hence the tension will probably exceed the ultimate within some equatorial belt. By definition of u, further opening of the fissure is then most probable at the equator but fluctuations of the stress can result in deflection of the path from the equatorial plane, or even the arrest of its progress. When the surface of the spheroid is not stationary, its equatorial rather than polar diameter expands. It also follows that secondary fissures may initiate in the highly strained equatorial belt,[12] their number being a matter of chance with small rather than large expectation within a short distance. The Poisson Law is clearly operative here, the expectation again being an increasing function of the strain energy. Let the time of a complete fissuring be denoted by $(\leq \tau)$. On the average, the source is equidistant from the boundaries. Defining a complete fissure as one which has produced two fragments the velocity of propagation is $v_f \geq L/2\tau$. By (50) (52) (58)

$$L/2\tau = L\dot{\rho}_n/\rho_n = \frac{L}{h}\sqrt{\mu/D}\Big/\frac{2uV_0}{kS_0}$$

or

$$v_f \geq \frac{3k\Delta}{uL}\sqrt{\mu/D}.$$

So long as the equator is expanding, the region ahead of the fissure experiences a radially advancing stress increment which moves with a velocity equal to or less than that of sound in the material. Therefore

$$\sqrt{(k + 4\mu/3)/D} \geq v_f \geq \frac{3k\Delta}{uL}\sqrt{\mu/D} > 0. \tag{80}$$

But Δ/L is arbitrarily small, within current limitations on technique. Therefore the velocity of the fissure may have any value less than that of sound.[11, 12]

This gives also a lower limit Δ_m on the size of the physical point:

$$h_m = L/\Delta_m = \frac{3k}{u}\sqrt{\mu/(k + 4\mu/3)}. \tag{81}$$

For samples of cast iron and Jena glass,[13] $u/3k = .0003$, $\mu = 4 \times 10^{11}$, $k = 5 \times 10^{11}$ dynes/cm.^{2}; whence $L/\Delta_m = 2000$. A usual absolute extension of tensile test specimens is $L = 20$ cm., and the properties in question were stated to one and two significant figures. The inhomogeneity of structure and strain would accordingly threaten the stability of fracture statistics on these materials for $\Delta_m \cong .01$ cm. The grain of cast iron is of this order of magnitude. Glass often shows initial stress inhomogeneities in regions larger than this. These results have the proper order of magnitude and tend to confirm our conclusions relative to the velocity of fracture propagation. Under static conditions, such as slowly increasing thermal strains, Hirata[12] has shown that the velocity of the fissures in glass sheets can be very small. The high velocity of fracture in weak brittle materials under crushing or shock loads is well-known. In the present instance we should expect high velocities to rule. A fissure will therefore tend to arrive at a given point at the same time as the accompanying redistribution of strain. Since new surface coincides with positive tension surface it follows that the prefracture strains largely determine the form of the new surface.

For random fracture the average form of new surface is given by the average strain. The surfaces of positive tension coincide with the strain potential surface. Integrating $\boldsymbol{\varrho}$ from $\mathbf{r}_0$ to $\mathbf{r}$ at time $t = \tau$, the difference of strain potential for the average solid is

$$\Phi(x, y, z, \tau) = \left[\frac{u}{6k}\mathbf{r}\cdot\mathbf{r} + \mathbf{r}\cdot\boldsymbol{\varrho}_0(\tau)\right]_{\mathbf{r}_0}^{\mathbf{r}}, \tag{82}$$

which is of the second degree. In the example of typical brittle material given above it was seen that *

$$u \ll 6k. \tag{83}$$

For materials of which this is true, Φ degenerates to a system of planes. $\boldsymbol{\varrho}_0(\tau)$ is an arbitrary constant displacement. Let it take unit value and the random orientation $\mathbf{n}$ of any plane in the system. The equation of the plane is then in the normal form. Let the second origin lie in a fissure plane and the vector $\mathbf{r} - \mathbf{r}_0 = \mathbf{n}_1 s$ denote the axis of the exploratory cylinder. Then

$$\mathbf{r}\cdot\boldsymbol{\varrho}_0\Big]_{\mathbf{r}_0}^{\mathbf{r}} = (\mathbf{r} - \mathbf{r}_0)\cdot\mathbf{n} = \mathbf{n}\cdot\mathbf{n}_1 s \tag{84}$$

is the equation of any plane in the system, $\mathbf{n}$ being its normal and $\mathbf{n}\cdot\mathbf{n}_1 s$ its normal distance from $\mathbf{r}_0$. The fissure spheroid has the axis $\mathbf{n}$ and the equatorial plane coincides with the

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* The ratio $3k/u$ might be termed the Brittleness of a solid.

tension plane. The tendency of the spheroid to expand with high velocity in this plane makes (84) the equation of new surface for the average solid.

The average value of $\boldsymbol{n}\cdot\boldsymbol{n}_1$ is 1, and of s is $\bar{s}$. But it is clear that the chance of sensible parallelism of a pair of random vectors $\boldsymbol{n}$, $\boldsymbol{n}_1$ is small. In fact, the chance of two vectors in physical coincidence is the ratio of the physically small solid angle $\Delta^2/L^2 s \left(\frac{\pi}{4}\right)^2$ to the total, or, that λ vectors do not coincide:

$$Q(\lambda) = 1 - \left(\frac{1}{4\pi h^2}\right)^{\lambda-1} \geq 1 - \left(\frac{1}{16\pi}\right)^{\lambda-1}\cdot \quad \lambda = 1, 2 \cdots \tag{85}$$

Hence any pair of opposite faces of the fragment are, on the average, parallel but the chance that they are sensibly out of parallel is more than 98 per cent. certain. The chance that three planes do not intersect is negligible. It is readily shown that they are on the average orthogonal. Again, the chance that sensible orthogonality occurs is small. Consider the physical point containing the intersection of three planes. The chance that n others intersect a radial vector $\boldsymbol{n}f\Delta$ from that point within the indefinitely small interval $f\Delta$ is by (1)

$$P_3(n) = \frac{1}{n!}(f\Delta/\bar{s})^n \frac{4\pi}{4\pi} = \frac{1}{n!}(q^{\frac{1}{3}}/\bar{s})^n, \tag{86}$$

since $f\Delta \ll s$. The chance that $n + 3$ planes intersect in a "sharp" physical point is therefore greater for much degraded, absolutely coarse materials and is a maximum for $n = 1$. It also becomes clear that $(n + 3)$-edge vertices are decreasingly probable as n increases. The chance that a fragment exhibits a concave vertex depends upon the improbable circumstance that two or more fissures arrive at the line of intersection simultaneously, at all points within an interval greater than Δ.

Summarizing the central tendencies of fragment shape, the average fragment is a plane, convex, equidimensional polyhedron. Its vertices are largely three-edged, its opposite faces parallel, and adjacent faces orthogonal; in short, a cube. The condition of high specific surface is also satisfied approximately by deviant forms, the 4, 5, 6 and 7 hedra, any of which can be produced by a single plane-cut through the cube. These also display three-edge vertices. The faces are 3, 4 and 5-sided polygons and these are the most probable fragments of a plane sheet. The hexagon and octahedron are unlikely in that they depend upon improbable vertex conditions. A *d*-sided fragment in *d* dimensions is impossible. The most probable number of sides in *d* dimensions is 2*d*. The possible number is infinite. These conclusions are most conveniently verified by counting vertices on fine-grained plane fragments, and preliminary experiments substantiate them.

The consistent classification of experimentally obtained shapes depends upon the stability of the precision of observation and the fineness of the material. Thus subjective estimates of shape show greater ambiguity for coarse sandstone than for glass fragments. This follows from the fact that the number (h) of subjective categories of position perceived in an extension L is highly variable, but affects less the chance of finding a given configuration for fine-grained materials with their sharp vertices. The ambiguity can be minimized by standardizing the sorting technique physically. The number of categories h to which a mechanism can be made sensitive is arbitrary within wide limits but, given this number and the material, its probable shape selectivity is determined.

EXPERIMENTAL: THIN BRITTLE ROD.

The thin cylindrical rod is the analytic element of the mechanically interpreted distribution function. It recommends itself as an experimental medium because strict isotropy of the material is not important, statistical homogeneity is easily attainable in large samples, the work of fracture is not large, and the fragments are easily classified. The question of shape is avoided since all fragments for which $s/\Delta > \sqrt{4/\pi}$ may be viewed as long cylinders. Shorter ones may be discarded as physical points, or, if retained, the discrepancies so introduced may be neglected provided that $\bar{s} \gg \Delta$.

The basis of the statistical reasoning is independent of the nature of the mechanism and may be tested in terms of geometrical fracture statistics without developing the energy correlations merely by disregarding the source of the material. However, the heterogeneity of a given distribution is immediately attributable to properties of the material and the strain, so that attention cannot long be averted from the underlying causes. Hirata[12] tested the Poisson Law for 800 similar thin, narrow, plane strips of glass subjected to similar flexure, by observing the number of fragments per strip and the principal lengths of the fragments. The fit was excellent. Hirata's careful control of conditions kept the problem of statistical heterogeneity of material and boundary forces in the background.

In this experiment the material was highly uniform, but the boundary conditions were as "haphazard" as could be devised. $1000 - 6.1'' \times .078''$ brittle spaghetti rods of circular section were inserted haphazardly in as many numbered, soft paper envelopes $4\frac{1}{8}'' \times 9\frac{1}{2}''$. Consecutive samples of 200 were divided into two piles, 100 high, to form a nearly cubical block. Capped by a block of soft wood the mass was placed between the heads of a vertically-acting 20,000 lb. testing machine and subjected to a single force deformation cycle. The period was several minutes and the force limits 0–1500 lbs. for the first 4 blocks. The last 200 were pressed to 3000 lbs. The force-deformation loop was integrated graphically as a rough index of the work W'. Except for a probable selectivity of position of the rod in the envelopes, this method appeared likely to partition the boundary forces and fracture work equally over the material and satisfy the velocity condition, although the process must be viewed as a concatenation of fracture cycles, all except the first finding the solids in a preliminary state of appreciable strain.

The material showed about 3^3 internal voids in Δ^3 and surface fluctuations of the same order. The starch grain was $f_2 = .0004$. The straightness was greater than L/Δ for $L = 6.1''$. The ultimate tensile strength by cantilever failure was 303 ± 30 lbs./sq. inch for 12 rods. From the same test Young's modulus was 7×10^6 lbs./sq. inch. Assuming that Poisson's ratio = .3; k and μ were 3×10^7. Whence $u/3k = 3 \times 10^{-6}$; $L/\Delta_m = 2 \times 10^5$. For $L = 6$ inches, the permissible smallness of the physical point is $\Delta_m = 3 \times 10^{-5}$ inches. The observed starch grain had an

TABLE I.
Statistical Parameters of Material.

u	Ultimate tensile stress (static test)	303 ± 30	lbs./sq. in.
μ	Rigidity	3×10^7	lbs./sq. in.
k	Bulk modulus	3×10^7	lbs./sq. in.
$u/3k$	Resistance to shattering	3×10^{-6}	—
L	Axial length of rod ($= 78\Delta$)	6.10 ± 0.04	in.
Δ	Diameter of rod and physical point	0.078 ± 0.003	in.
$f_1\Delta$	Surface irregularities and void diameter	0.003———	in.
—	Voids per physical point	30	—
L/Δ_m	Calculated maximum physical points along L	2×10^5	—
Δ_m	Calculated smallest permissible physical point	3×10^{-5}	in.
$f_2\Delta$	Observed approx. diameter of finest grain	2×10^{-5}	in.
l	Estimated displacement of fracture	2×10^{-5}	in.
m	Number of rods per sample	100	—
$\alpha_{1,8}$	Number samples in comparable group $r = 1$–8	8	—
$\alpha_{9,10}$	Number samples in comparable group $r = 9, 10$	2	—
$\alpha_{11,12}$	Number samples in comparable group $r = 11, 12$	2	—
—	Number of all rods	1200	—

approximate diameter 2×10^{-5} inches. This indicates that the smallest observable structure is the quantum of these statistics and that prediction would become relatively uncertain for $s = 10^{-5}$ inches if the material were ground finer than the physical point, 0.078^3 inch.[3] See Table I and Fig. 4.

The fragments in each envelope were measured and counted, discarding small pieces less than 0.078″ in length and/or not completely cylindrical. The fractures per rod were found to agree with $P(n)$ as Hirata found. This method of test involves extensive tabulation and is not as convincing intuitively as the oversize frequency distributions. If the condition of homogeneity is not satisfied, the found oversize function need only be monotonously decreasing from $s = 0$ to $s = L$. If no rods are broken a rectangular distribution must result. For many fractures per rod and many rods under heterogeneous conditions, the one-termed exponential is approached with a minor discrepancy for $s \leqq \Delta$. The short cylinder, if it fractures at all, improbably produces other cylinders upon fracture. With these discrepancies noted in advance the departures of the found oversize distributions from the simplest form of the expectation are readily intelligible.

TABLE II.

Found Proportions of Fragments Oversize.

Sample r.	Size $s/\bar{s}_{mr}'$.	Proportion Oversize*.	Discrepancy†.	Sample r.	Size s/s_{mr}'.	Proportion Oversize*.	Discrepancy†.
1	0.00	1.000	.000	4	0.00	1.000	.000
	0.41	.521	−.143		0.42	.547	−.107
	0.82	.403	−.038		0.85	.395	−.033
	1.23	.289	−.004		1.27	.267	−.014
	1.64	.196	.001		1.70	.199	.015
	2.05	.140	.011		2.12	.135	.015
	2.45	.094	.008		2.54	.101	.021
	2.86	.078	.021		2.97	.081	.030
	3.27	.064	.026		3.39	.058	.024
	3.68	.052	.027		3.81	.033	.011
	4.09	.042	−.015		4.24	.029	.015
	4.50	.034	−.033		4.66	.019	.010
2	0.00	1.000	.000	5	0.00	1.000	.000
	0.28	.626	−.128		0.44	.587	−.059
	0.56	.511	−.058		0.87	.387	−.030
	0.85	.408	.000		1.31	.253	−.017
	1.13	.319	.011		1.75	.173	−.001
	1.14	.247	−.057		2.18	.124	.009
	1.69	.281	.097		2.62	.097	.024
	1.97	.181	.042		3.06	.071	.024
	2.26	.123	.018		3.50	.043	.014
	2.54	.101	.022		3.93	.032	.012
	2.82	.081	.021		4.37	.017	.004
	3.10	.075	.030		4.81	.017	.009
3	0.00	1.000	.000	6	0.00	1.000	.000
	0.30	.684	−.058		0.47	.528	−.097
	0.60	.512	−.040		0.94	.348	−.042
	0.89	.397	−.012		1.41	.246	.002
	1.19	.303	−.001		1.88	.157	.004
	1.49	.231	.006		2.35	.113	.018
	1.79	.179	.012		2.82	.075	.015
	2.09	.140	.016		3.29	.056	.019
	2.38	.110	−.017		3.10	.045	.000
	2.68	.091	.022		3.41	.033	.000
	2.98	.061	.010		3.58	.028	.000
	3.27	.050	.012		3.65	.026	.000

TABLE II (*Continued*).

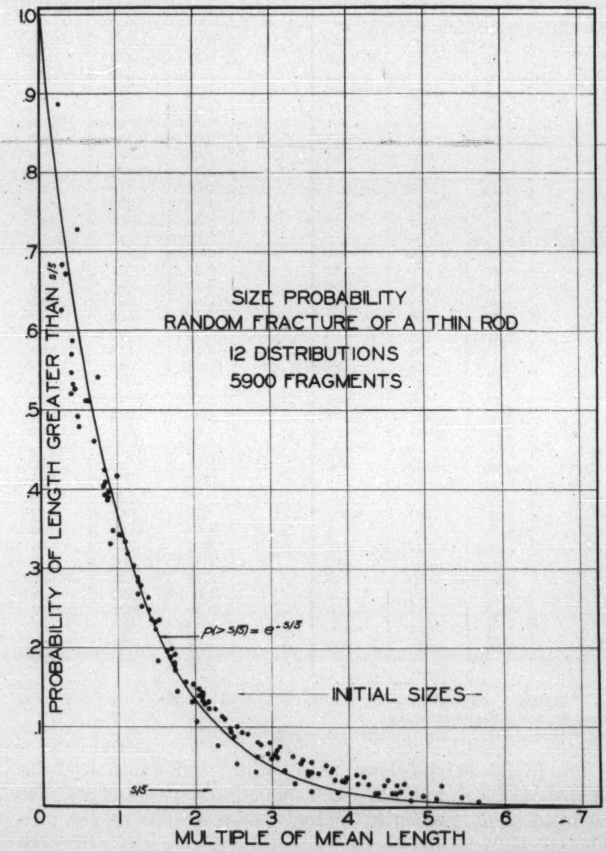
Sample r.	Size $s/\bar{s}_{mr}'$.	Proportion Oversize*.	Discrepancy†.	Sample r.	Size $s/\bar{s}_{mr}'$.	Proportion Oversize*.	Discrepancy†.
7	0.00	1.000	.000	10	0.00	1.000	.000
	0.50	.949	.339		0.51	.478	−.121
	0.99	.330	−.042		1.03	.342	−.016
	1.48	.223	−.004		1.54	.235	.020
	1.98	.155	.017		2.05	.149	.020
	2.47	.095	.011		2.56	.101	.024
	2.97	.070	.019		3.07	.062	.016
	3.46	.046	.015		3.59	.042	.014
	3.96	.036	.017		4.10	.029	.012
	4.45	.026	.014		4.61	.016	.006
	4.95	.013	.006		5.12	.010	.004
	5.44	.011	.007		5.64	.006	.002
8	0.00	1.000	.000	11	0.00	1.000	.000
	0.42	.570	−.087		0.35	.671	−.044
	0.84	.426	−.006		0.70	.462	−.034
	1.26	.285	.001		1.05	.342	−.008
	1.68	.186	−.001		1.40	.263	.017
	2.10	.133	.011		1.75	.192	.019
	2.52	.088	.008		2.10	.143	.021
	2.94	.066	.013		2.45	.098	.012
	3.36	.055	.020		2.80	.062	.001
	3.78	.048	.025		3.15	.045	.002
	4.20	.037	.022		3.50	.024	−.006
	4.62	.027	.017		3.85	.019	−.002
9	0.00	1.000	.000	12	0.00	1.000	.000
	0.43	.533	−.115		0.25	.888	.112
	0.87	.378	−.042		0.51	.728	.125
	1.30	.278	.005		0.76	.541	.073
	1.73	.180	.003		1.01	.417	.054
	2.16	.132	.018		1.27	.284	−.002
	2.60	.093	.019		1.52	.183	−.036
	3.04	.066	.018		1.77	.145	−.025
	3.22	.040	.000		2.02	.107	−.026
	3.44	.032	.000		2.28	.077	−.025
	4.07	.017	.000		2.53	.053	−.027
	4.66	.009	.000		2.78	.044	−.018
					3.29	.030	−.007

* Precision of proportions not justified in light of sampling error but permits recalculation of fragment count.

† Discrepancy = found proportion oversize $-e^{-s/\bar{s}}$.

The found distributions are given in relative oversize form (Table II) for samples of 100. A preliminary set of 200 also included were crushed by laying known weights on the pile. The geometrical and mechanical statistics are given for each sample of 100. The work done on a mass of 200 was imputed 50 : 50 to the component hundreds. The given parameters and proportions permit recalculation of the numbers falling

FIG. 1.


SIZE PROBABILITY
RANDOM FRACTURE OF A THIN ROD
12 DISTRIBUTIONS
5900 FRAGMENTS.
PROBABILITY OF LENGTH GREATER THAN s/s̄
MULTIPLE OF MEAN LENGTH
p(>s/s̄) = e^{-s/s̄}
INITIAL SIZES
s/s̄
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
1
2
3
4
5
6
7

into the 12 size categories allowed in classification of the fragments. This classification was effected by a wedge gauge whose absolute error per trial was less than a rod diameter. All distributions are plotted to the homogeneous scale $s/\bar{s}$ in Fig. 1. The discrepancies due to initial sizing and the "short" cylinder effect are apparent. These are larger than random sampling of a homogeneous universe would lead us to expect. See Table III.

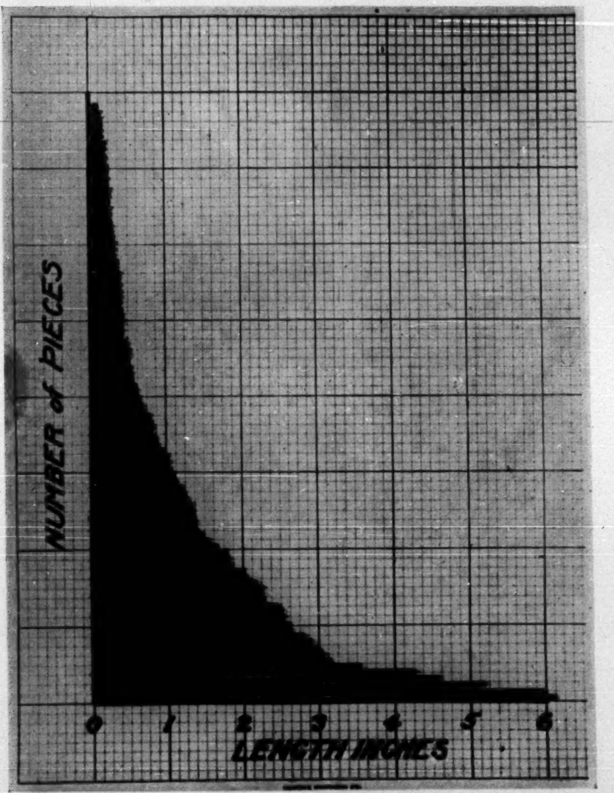
A mechanical model for randomness is obtained by sampling a universe of fragments for any number of pieces without bias as to length. The longer fragments are laid parallel to and nearer the axis on a piece of rectangular coördinate paper. The left ends coincide with $s = 0$. The number of rods intersected by an ordinate at s is $\nu(> s)$. This has been done in Figs. (2–3). The smooth curve was calculated from $\bar{s}$ of the universe and N_0 of the sample. The

TABLE III.
Distribution Parameters.

Sample.	Components of Sample.	Maximum Crushing Force, Lbs.	Work of Fracture,* Inch Lbs.	Number of Fragments	Total Length, Inches.	Mean Fragment, Inches.	±Sampling Limits of Mean, Inches	New-Surface Energy Inch Lbs.†	Maximum† Efficiency	Mean‡ Discrepancy of Found from $e^{-s/\bar{s}}$	Mean Absolute Discrepancy‡
r.	$i-j$.	–	W'.	N_{0r}'.	mL.	$\bar{s}_{mr}'$.	$\delta\bar{s}_{mr}'$.	$W \leqq$.	$\epsilon \leqq$.	–	–
1	1– 100	750	195	449	610	1.224	.055	2.5	.013	.015	.027
2	101– 200	750	195	348	610	1.753	.094	1.7	.009	.000	.040
3	201– 300	750	245	363	610	1.680	.088	1.8	.007	−.001	.017
4	301– 400	750	245	517	610	1.180	.052	2.6	.011	−.001	.025
5	401– 500	750	245	533	610	1.145	.050	2.7	.011	−.001	.017
6	501– 600	750	245	574	610	1.063	.044	2.9	.012	−.007	.016
7	601– 700	750	209	611	610	1.011	.040	3.1	.015	.033	.041
8	701– 800	750	209	512	610	1.191	.053	2.6	.012	.019	.018
9	801– 900	1500	373	529	610	1.153	.050	2.6	.007	−.008	.018
10	901–1000	1500	373	625	610	0.976	.039	3.1	.008	−.002	.021
11	1001–1100	180	(195)**	468	668	1.427	.066	2.3	.012	−.002	.014
12	1101–1200	180	(195)	338	668	1.976	.107	1.7	.009	.014	.041

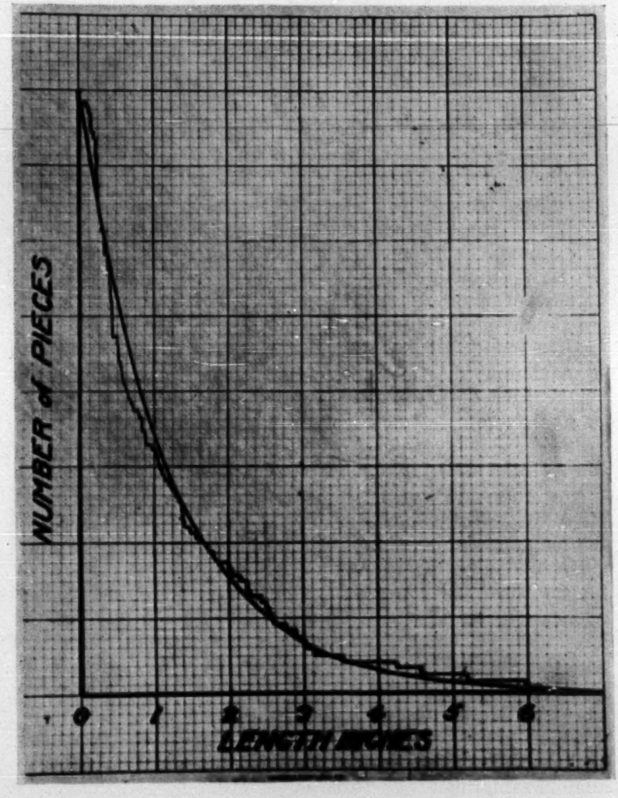
* (Per Block of 200) ÷ 2.
† Assuming $l \leqq 2 \times 10^{-5}$ inches; $\epsilon = W/W'$.
‡ Unweighted means.
** Estimated.

FIG. 2.



Model of a linear random fracture series. Sample of 100 fragments from series No. 1. See comparison with curve of universe, Fig. 3.

FIG. 3.



Fracture model compared with expectation $\nu(>s) = 100e^{-s/1.224}$. Calculated from series No. 1.

photograph shows the expected preponderance of normal fracture surfaces, the "short cylinder" effect, and the presizing discrepancies. This distribution is readily produced by crushing "haphazardly" with the hands any fairly uniform thin rod. The writer finds this operational example of Poisson's "Law of Small Numbers" intuitively more satisfying than Bortkewitsch's classic horse-kick example.

The fracture displacement l is estimated as $\leq 2 \times 10^{-5}$ inches, in keeping with the finest observed structure $f_2\Delta$ and the predicted smallest physical point Δ_m. The efficiency on this basis averages $\epsilon \leq 0.012$ which is comparable with 0.003 calculated from Hirata's data on thermally induced fracture of glass. Interest does not center on the actual magnitude of ϵ so much as its smallness. The correlation between s and W' is, of course, positive and in view of its rational basis and significant magnitude, 0.6 ± 0.3, could serve with ϵ as a basis for predicting s if the experiments were repeated.

The found distributions are monotonously decreasing with a maximum at $s \geqq \Delta$. The assumption that $N_i \gg 1$ was not realized here, for there were only 4.9 fragments per rod on the average. A vestige of the original rectangular distribution should therefore probably remain in the vicinity of the initial sizes. Sample No. 10 with its short mean length is the only one showing a persistent negative discrepancy from $e^{-s/\bar{s}}$. In general, the discrepancy is positive and this in the range of pre-sizes. The sampling tests show that a slight inhomogeneity of fracture conditions existed, apparently for the most part due to the selectivity of the mechanism. Thus the rods were cut to fit more or less haphazardly into the envelopes. There was, nevertheless, a preferential effect here and the vestige of presizing, and selective fracture due to position in the envelope may be assumed to account for 25 per cent. of the observed dispersion of means. Of the remainder, 50 per cent. is attributed to the material on the basis of the tenacity $u = 303 \pm 30$ lbs./sq. in., and 25 per cent. to random sampling. See Table IV.

A better fit can be obtained by retaining more terms of the exponential series eq. (11), but these are here not very significant in view of the large sampling errors of the means. The present aim has been to throw the heterogeneities into relief and show that despite the small amount of new surface and selectivity of the crushing mechanism, the simplest theoretical distribution is rapidly approached.

TABLE IV.

Homogeneity Parameters.

h	Precision of fragment classification	≤ 78
$\bar{s}_{m\alpha}'$	Mean of sample means (1 . . . 8)	1.282″
$\delta\bar{s}_{m\alpha}'$	Mean of sampling limits (1 . . . 8)	.060″
σ	Dispersion of sample means (1 . . . 8)	.260″
$\delta/\bar{s}_{m\alpha}'$	Relative sampling limits (1 . . . 8)	.05
$\sigma/\bar{s}_{m\alpha}'$	Relative dispersion means	.20
—	Sampling chance of this or larger dispersion	$<.001$
—	Dispersion attributable to sampling	$.25\sigma$
—	Dispersion imputed to material (see u)	$.5\ \sigma$
—	Dispersion to selectivity of fracture process	$.25\sigma$
ϵ	Mean efficiency (1 . . . 12) *	$<.012$
—	Correlation—New surface—Gross work (1 . . . 12)	$.6 \pm .3$
—	Proportions showing negative discrepancy	44
—	Proportions showing positive discrepancy	80
—	Proportions showing zero discrepancy	20
—	Mean † observed discrepancy from $e^{-s/l}$	$+.005$
—	Mean absolute discrepancy from $e^{-s/\bar{s}}$	.025

* Compare Hirata, $\epsilon = .003$ for thermally produced cracks.[12]

† Unweighted.

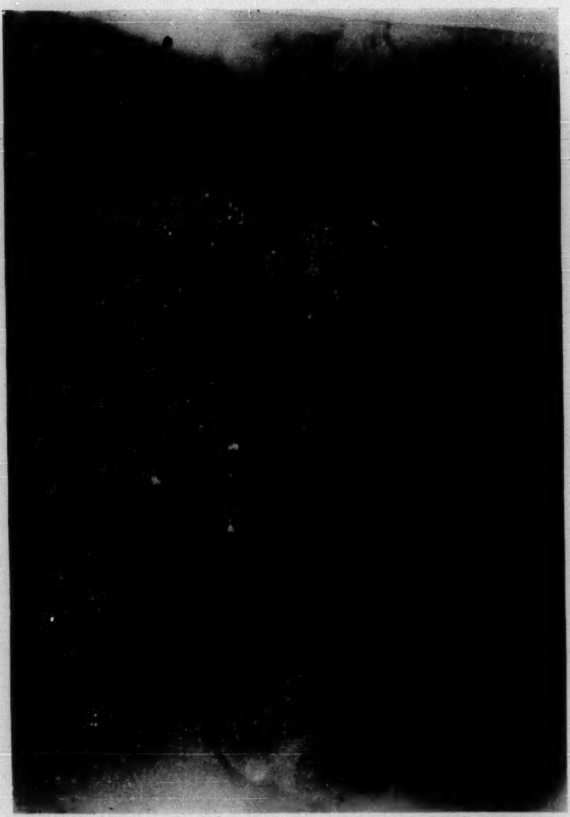
SUMMARY.

This paper has applied a tentative definition of a statistically homogeneous brittle solid in (*a*) the determination of the average regional and boundary conditions which accompany the production of a random system of new fracture surface, and (*b*) the calculation of the size, mass and shape probabilities of the resulting system of fragments.

(*a*) Brittleness is an intensive property of a solid which depends among other things upon the temperature. A common conception of a brittle solid as one whose reaction to shock is the production of new surface by shattering is made more precise by measuring the resistance to shattering by $u/3k$, where u is the ultimate static tension and k the bulk elastic modulus. For samples of glass and cast iron the resistance to shattering is 3×10^{-4}, and for the material used in the present experiment 3×10^{-6}.

A statistically homogeneous brittle solid free from hysteresis and obeying Hooke's Law to the fracture point is assumed to fail only by parting along surfaces where the average positive tension becomes u. The solid contains no internal

FIG. 4.



Rod cross-section showing surface fluctuations, internal void, and finest observed structure.

voids larger than the " negligible " fragment and has a convex surface, but is otherwise arbitrary in form and volume. It may be represented by an equivalent sphere. The average regional condition for a random distribution of initiating fractures is a homogeneous positive tension u. The boundary conditions prior to fracture can be satisfied by two or more slowly acting members of a random system of point forces which do not disturb the average position of the center of gravity while producing deformation at a constant time rate. The average work of the boundary forces prior to fracture is that necessary to produce a homogeneous tension u in the average solid. This assumes that there are no organized strains prior to the application of the boundary forces. The only restriction on the actual work done on the solid by the boundary forces in the production of random initiating fracture is that it be positive.

(*b*) The energy balance for the production of fracture surface in the average solid contains terms representing the work done by the boundary forces, the energy made available by the relaxation of initial random strains, that entering the production of new surface, and that entering all other categories. A special form of the Second Law asserts that the new fracture surface of a brittle solid is either increasing or stationary within the period of deformation and fracture. When fracture occurs, a stationary state is reached in a time short compared with the prefracture period. The assumption of a discrete fine structure and the randomness of new surface serve to connect the effective boundary work per unit volume and the tensile strength with the " random fragment-diameters." The concept of the thin rod segmented at random by the fracture surface is basic in establishing the Poisson Law as the sought distribution function. The size and mass distributions for homogeneous and heterogeneous systems are then readily calculable in terms of statistical analysis developed independently of the mechanical setting. The usual statistical tests for homogeneity, and the special test constructed here, become applicable in judging the homogeneity of the material and force system. In general, a heterogeneous fracture system is capable of description by a finite series of exponential terms. In the case of numerous components, none of which predominates, a single exponential term is approached. The average shape of the fragment is cubical. This follows from the above mentioned condition that the new surface tends to a maximum.

The initial average elementary fissure departs from the sphere due to inhomogeneity of structure and strain at that level. Once formed the spheroid is most probably enlarged in the equatorial plane with a velocity which is equal to or less than that of sound in the material. The equatorial plane coincides with a surface of positive tension. For the average solid the positive tension surfaces are random planes. Random planes intersect a directed line at random and have random orientation. The mean perpendicular distance from a random origin in the system of planes to one such plane is a statistical parameter of fracture. The new surface distribution can be described by a linear function of the coördinates, the Poisson Law for random linear segments and the orientation probabilities of the sheaf of random unit vectors about a point. The probabilities of various fragment forms deviating from the cube can be calculated on this basis. In d dimensions the $2d$-sided fragment is most probable, equi-dimensionality and irregularity being the rule. For consistent experimental classification of form, adherence to a particular order of discrimination is required, the " sharpness " or ambiguity of a vertex being a function of the observer's acuity as well as the fineness of structure of the material.

The experimental work deals with the simple but fundamental case of one dimensional fracture. The aim of the experiment was to illustrate the operation of the concepts basic to the analysis in a situation involving a minimum of irrelevancy. It was shown that for a fairly uniform thin brittle rod and a considerably disordered set of boundary forces, the random distributions anticipated with the aid of the Poisson Law are rapidly approached. Discrepancies are reasonably accounted for as the result of observable inhomogeneities of the material and an incomplete lack of selectivity of the boundary conditions. The so-called efficiency of fracture is low.

Evidence for the applicability of the present analysis is found in the empirical studies of Martin, Gaudin, Suzuki, Terada and Hirata on the fracture statistics of solids. There are noted some possible contacts of this analysis with a point of view less special than proved expedient in the case of the brittle solid.

April 9, 1935.

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New Marathon Plant to Begin Operations.—(*Paper Trade Journal*, Vol. CII, No. 4.) As a result of five years of experimenting and research a new venture will be started by the Marathon Paper Mills Co. of Rothschild, Wis., in the form of the first complete plant for the processing of waste sulphite liquor for the double purpose of reclaiming chemicals and pulp substances either for re-use or commercial marketing, and the elimination of the much contested right of paper mills to cause stream pollution. The new plant covers an area of 20,000 sq. ft. and is located adjacent to the acid plant and boiler house of the Marathon mills. Waste sulphite liquor collected from the blow-pits will be treated by the Howard process, a precipitation treatment with lime. This yields a lime-sulphur product for use by the pulp mill in making fresh cooking acid, a lignin product for use by the mill as a boiler fuel or to be utilized as a chemical raw material for further processing into various lignin products, a process effluent of sufficiently improved character to permit discharging into the Wisconsin River without danger to pollution, and a number of incidental savings to the pulp mill. Patent rights are owned jointly by the Marathon Paper Mills Co. and Guy C. Howard, consulting chemical engineer, who carried on the development work.

R. H. O.

Photographing Shaft Interiors by Reflected Sunlight.—W. W. FLEMING, of the U. S. Geological Survey, describes in *Mining and Metallurgy*, Vol. 17, No. 350, how reflected sunlight photography is used to show details of walls and timbers in mine shafts in the McAlester coal mining district, Oklahoma. Entrances to many shafts and slopes are being filled or bulkheaded and it was suggested that the photographs might be of value, inasmuch as these mines may be reopened at some future time when there is a demand for coal. Artificial illumination for such work gave varying results, more or less unsatisfactory. With the use of reflected sunlight a marked improvement has been obtained. When taking pictures the camera is fixed in one location, with the aperture set at about F32 and the lens shutter opened. Sunlight is then reflected by a common, unbeveled mirror. The person holding the mirror manipulates it by moving it in a circular movement so that the reflected light sweeps around all the walls of the shaft or slope, flooding them with bright light. The procedure can be described as "sunshine painting." During this time, of course, the camera is open and the process of photographic recording is going on. From one to two minutes is required for a good exposure, although the time varies with conditions.

R. H. O.

THE COEFFICIENT OF FRICTION FOR YARNS.

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SUMMARY.

In view of the importance of the frictional characteristics in the manufacture of yarns and in the final appearance of textile products, a method has been devised with which it is possible to determine the coefficient of friction of individual yarns in absolute figures.

It consists substantially in paralyzing the constant, rotary moment of an unbalanced wheel by the frictional resistance of a piece of yarn, loaded at each end with a definite weight. The yarn is in contact with one-half of the carefully standardized surface of the rim of the wheel. The mathematical analysis of the physical principle involved is given.

The method described lends itself also to the investigation of the frictional characteristics of different weaving and knitting constructions, to the determination of the lubricating value of different throwing oil compounds and various oils, the influence of oxidation upon the lubricating value of oils, the change in the friction of yarns imparted to them by sizes and other agents.

The coefficients of friction for several different types of yarns and conditioned yarns have been tabulated.

INTRODUCTION.

One of the factors that influences the production of yarns, cloth and knitted goods is the friction of the individual yarn. It is the coefficient of friction, μ, that determines what tension is appropriate during quilling or in the shuttle for giving the filling adequate resistance. Variations in yarn friction necessitate changes in the tension adjustments. If the surface of the yarn is rough, the tension weight must be reduced while for smooth, well oiled yarn, the tension has to be increased.

Accordingly, the tension necessary and permissible during the drawing, warping and other operations varies with the coefficient of friction of the individual yarn. Furthermore, the frictional properties also influence the "handle and feel" of the finished cloth. Last but not least, the change in the coefficient of friction of yarn as imparted to the yarn by different throwing oils, sizes and other conditioning compounds is a factor, the quantitative knowledge of which is necessary in research as well as in the efficient production which is based on exact knowledge rather than rule of the thumb methods. The correct setting of the tension controls has to be in correspondence with the absolute, numerical values of yarn friction.

A method has been developed, therefore, for determining objectively this important physical characteristic of textile materials: the coefficient of friction, μ. While methods were known before for estimating *qualitatively* the frictional value of yarns[1] and whole skeins in relation to each other, the principle described hereafter permits the *quantitative* determination of absolute numerical figures of the coefficient of friction for single individual yarns as well as for yarn combinations. The qualitative methods referred to may be calibrated against this absolute method.

The rigid theoretical proof given lends itself in an analogous manner for the determination of μ as well for wires such as tungsten filaments, in weaving- and knitting-constructions as for any other type of combination of yarn with yarn, or yarn with other materials, such as lubricants. To consider extreme applications of the method: cloth for brake linings could be developed for maximum frictional characteristics with an apparatus based upon the same principle or the difference in the lubricating characteristics of various throwing oils can be determined for different yarns and also the optimum amount of total size to be deposited upon the yarn for best results. This lubricating efficiency is not constant as such but has been found to change with the age of the conditioned yarn, the degree of oxidation of the throwing oil compounds, etc.

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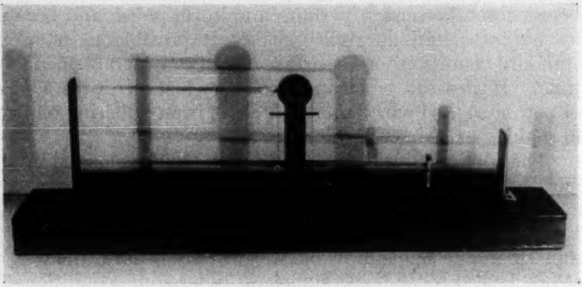
[1] W. B. Sellars, "Yarn Friction Tests," *Textile World*, Dec. (1934), pp. 2408. John H. Skinkle and R. C. Morrison, "Lubricating Power of Oils," *American Dyestuff Reporter*, Vol. XXIV, No. 2, Jan. (1935). Mercier, *American Wool and Cotton Reporter*, Nov. 13, 1930. *American Silk and Rayon Journal*, Vol. LIV, No. 10, Oct. (1935), p. 32.

Furthermore, the influence of viscosity of oils and their total amount deposited upon the yarn in relation to μ, their proper combination with binders such as gelatines or starches can be studied with this method.

DESIGN OF THE INSTRUMENT

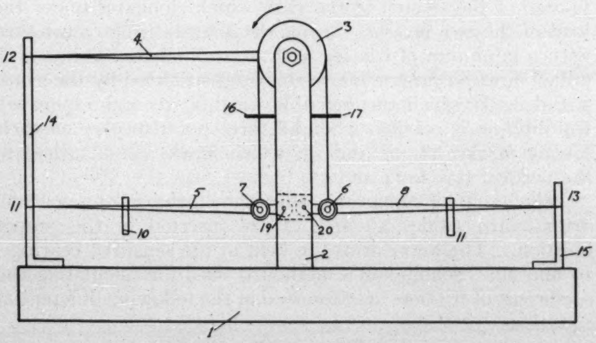
Figure 1 is a photograph of the Friction Tester. The principle involved will become clear by studying Fig. 2, in

FIG. 1.



The friction tester.

FIG. 2.



Constructional diagram of the friction tester.

which the construction details are depicted diagrammatically.

Attached to the base plate (1) is a vertical support (2). This stand holds a carefully machined and uniformly chromium plated wheel (3), in a balance suspension. This wheel tends to turn in the direction of the arrow on account of the aluminum arm (4) which is securely fastened to it. This arm is designed in such a manner that a force of 30 grams applied tangentially at the periphery of the wheel is necessary to overcome the torque of the lever. To ascertain a standard surface of the wheel between which end the yarn the friction takes place, the wheel has been carefully machined and chromium plated thereafter. Chromium has been chosen because it permits a uniform polish and because its hardness and durability makes it possible to maintain standard conditions of its surface over a considerable period of time. It also can be cleaned well (for instance with a piece of cotton soaked with ether) to free it from adhering oil used in soaking experiments.

The frictional resistance of the yarn which is loaded by the movable weights upon the arms, 5 and 8, acts against the moment of rotation imparted to the wheel, 3, by the arm, 4, which latter is securely attached to the wheel. In practice sometimes one single movable weight only is used, the weight of one lever proper being sufficient under some conditions.

The yarn itself is fastened to the levers by the binding post, 7, and the turnable peg, 6. The latter permits the takeup of the stretch of the yarn which elongates under the load of the two levers. Again, the lever bearings must turn with a minimum of friction and are well oiled. The yarn is pulled down in proportion to the weight exerted by the arms, 5 and 8, on which the movable weights, 10 and 11, travel. Equilibrium is reached when all three pointers play on their setting marks, 11, 12 and 13, which marks are attached to the vertical stands 14 and 15.

For avoiding transverse slippage of the yarn upon the drum, yarn guides 16 and 17 are inserted in the proper position. The lower arms are held in the sensitive bearings, 19 and 20. Sokolnikoff's method is used for calculating the coefficient of friction,[2] as expressed in the following differential equations.

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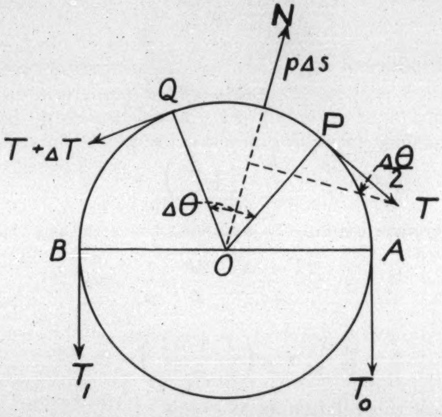
[2] I. S. and E. S. Sokolnikoff, "Higher Mathematics for Engineers and Physicists," p. 214–216; McGraw-Hill Book Company (1934).

THEORY.

Let T_0 and T_1 be the tensions of the yarn (Fig. 3), at the points A and B.

Consider an element of length Δs of the yarn which subtends an angle $\Delta\theta$ at O, with end points P and Q. Let the tension at P be T, and at Q be $T + \Delta T$, and let the normal pressure per unit of length of the arc be p, so that the total normal force on the element of the arc Δs is $p\Delta s$. If the angle $\Delta\theta$ is assumed to be small, the normal pressure may be thought of as acting in the direction of the line ON.

FIG. 3.



From the definition of the coefficient of friction μ, the frictional force is equal to the product of μ by the normal pressure, so that the frictional force on PQ is $\mu p\Delta s$. Since $\Delta\theta$ is small, this frictional force may be assumed to act at right angles to ON. If it is assumed that the yarn is at the point of slipping, the components of force along ON must balance. Hence:

$$T \sin\frac{\Delta\theta}{2} + (T + \Delta T)\sin\frac{\Delta\theta}{2} = p\Delta s$$

or

$$(2T + \Delta T)\sin\frac{\Delta\theta}{2} = p\Delta s. \qquad (1)$$

Similarly, by equating the forces acting at right angles to ON,

$$(T + \Delta T) \cos \frac{\Delta\theta}{2} - T \cos \frac{\Delta\theta}{2} = \mu p \Delta s$$

or

$$\Delta T \cos \frac{\Delta\theta}{2} = \mu p \Delta s. \tag{2}$$

Eliminating $p\Delta s$ between (1) and (2) leads to

$$\frac{2T + \Delta T}{\Delta T} \tan \frac{\Delta\theta}{2} = \frac{1}{\mu}. \tag{3}$$

For small values of $\Delta\theta$,

$$\tan \frac{\Delta\theta}{2} = \frac{\Delta\theta}{2}$$

since

$$\tan \frac{\Delta\theta}{2} = \frac{\Delta\theta}{2} + \frac{1}{3}\left(\frac{\Delta\theta}{2}\right)^3 + \cdots,$$

so that (3) approximates

$$\frac{2T + \Delta T}{\Delta T} \cdot \frac{\Delta\theta}{2} = \frac{1}{\mu}$$

or

$$\frac{\Delta T}{\Delta\theta} = \mu \left(\frac{T + \Delta T}{2} \right).$$

The limit of this relation as $\Delta\theta \to 0$ is

$$\frac{dT}{d\theta} = \mu T.$$

Upon separation of the variables this becomes

$$\frac{dT}{T} = \mu d\theta$$

and integrating gives

$$\log T = \mu\theta + c$$

or

$$T = c_1 e^{\mu\theta}. \tag{4}$$

The arbitrary constant c_1 which enters into the solution of the differential equation can be determined from the initial condition $T = T_0$ when $\theta = 0$. Substituting these values in (4) gives, $T = T_0 e^{\mu\theta}$ so that the tension T_1 corresponding to the angle of the lap is

$$T_1 = T_0 e^{\mu\alpha} \tag{5}$$

The angle α is determined in radians. It is in our case π degrees. The equation (5) becomes therefore

$$T_1 = T_0 e^{\mu\pi} \quad \text{or} \quad \frac{T_1}{T_0} = e^{\mu\pi} \tag{6}$$

and finally,

$$\mu = \frac{\ln T_1 - \ln T_0}{\pi}. \tag{7}$$

With the aforementioned method the friction has been determined for several yarns. The following table contains the coefficients of friction for a number of yarns that are used in the textile industry.

Coefficients of Yarn Friction.

Each figure represents the average of 5 independent measurements.

Type of Yarn.	T_1.	T_0.	$\mu = \frac{\ln T_1 - \ln T_0}{\pi}$.
Rayon 200/80 (natural)	90	61	0.126
Rayon 200/80 (plus conditioning oil)	103	76	0.101
Acetate 200/64	88	56	0.144
Acetate 150/50	85	56	0.135
Spun Rayon 1/22	111	88	0.0536
Sized Acetate 100/40	85	57	0.127

CONCLUSIONS.

1. The coefficient of friction for synthetic yarn is comparatively low. A μ of 0.126 as is the case with viscose type yarn is only a fraction compared to the friction of leather on dry metals with a μ of about 0.56.
2. The coefficient of friction in spun yarns is particularly low.[3]
3. Conditioning reduces yarn friction.
4. The friction of acetate type yarns is generally higher than the friction of viscose type yarns.

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[3] This is one of the reasons why it is often possible to spin and weave rayon staple without slashing.

Platinum, Palladium, Rhodium and Iridium in 1935.—CHARLES ENGELHARD. (*I. and E. C.*, News Edition, Vol. 14, p. 29.) In the chemical industry, platinum enjoyed continued use as a catalyst and in laboratory equipment. The possibility of platinum-clad chemical apparatus is nearing more and more a reality. A flute of iridium-platinum commanded international attention. It was demonstrated for both musicians and scientists by Georges Barrere, noted flutist, and is claimed to have greater purity of tone and ease of performance than any flute so far made. Platinum has become well established in the rayon industry where platinum-gold spinnerets are used with caustic solutions. In the electrical field, platinum points in magnetos provide a high degree of reliability for airplane motors.

Rhodium found increasing use as a finish for reflectors and for the protection of silverware from tarnish. The "rhodanizing" of decorative silver pieces is being offered as a service by leading department and jewelry stores throughout the country.

Palladium leaf is finding wider application in the decorative art. It provides a white leaf in contrast to the yellow of gold and a method for applying colored pigment to the metal surface has been developed.

C.

HIGH SPEED ELECTRIC MOTORS.

BY

R. C. COLWELL and N. I. HALL,

Department of Physics, West Virginia University.

SUMMARY.

In this paper, two new types of electric motors are discussed which are capable of rotating 100,000 r.p.m. A synchronous motor is described but the emphasis is placed upon the new design of the induction motor. This motor is mounted upon a single bearing and for high speeds must be run in a vacuum.

The high frequency current (3,000 r.p.s.) is produced from a two phase Alexanderson alternator, which is described. The circuit of this alternator must be tuned to resonance. The velocity of the rotating motor is measured with a new type of stroboscope which has a wide range of values and is easily calibrated.

Aluminum alloys for the disks and commercial bearings have been tested and found suitable with speeds up to 75,000 r.p.m.

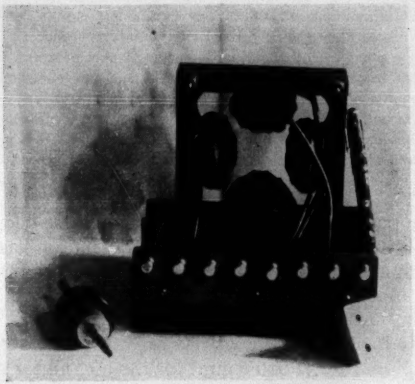
In the past, very high rotative speeds have been obtained mainly with air turbines. The electric motor has not been used much at high speed because of commutation troubles in the d.c. motor and lack of a convenient high frequency alternating current supply for the a.c. motor. The highest motor speeds of which the writers are aware are 20,000 r.p.m. for the d.c. motor, 32,000 r.p.m. for the a.c. motor and 60,000 r.p.m. for the air turbine. Also, Professor Beams of the University of Virginia has driven an air supported centrifuge at 1,200,000 r.p.m.

Experiments are being made at West Virginia University to determine the possibilities of various types of electric motors for getting very high speeds and to develop simplified means for obtaining the necessary high frequency alternating current to drive them. All experiments have dealt with commutatorless a.c. motors as they appeared to have the greatest possibilities.

The first type of motor developed was the synchronous motor having no d.c. field winding and therefore no slip rings or rotor winding. The highest speed at which this motor has been run is 45,000 r.p.m., although it can be run at a somewhat higher speed without flying apart.

Four major problems were encountered in the construction and operation of the synchronous motor and they also occur in any high speed motor. First, a stable supply of high frequency alternating current; second, the dynamic balance of the rotating part; third, the mechanical strength of the rotor; fourth, a bearing which will stand the terrific speed. How these problems were met will be shown in the description of the motors.

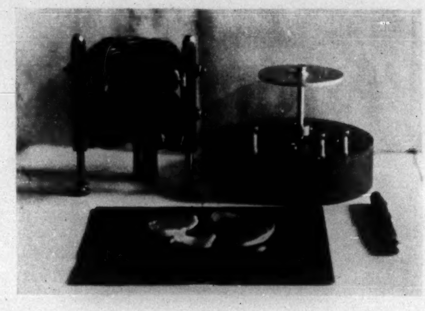
Fig. 1.



High speed synchronous motor.

In its final form the motor was constructed as shown in Fig. 1. It is connected as a two pole two phase motor. The rotor is made of a stack of steel laminations which were ground into the shape shown in the diagram. The grinding was done as accurately as possible, but in order to obtain a more perfect balance all of the laminations were removed from the shaft, shuffled and replaced at random. This method worked quite well and no vibration was noticed even at the highest speed at which the motor was run, i.e., 45,000 r.p.m. The only satisfactory source of power available was a 500 cycle inductor alternator which was run fifty per cent. over speed to obtain 750 cycles. The motor was synchronized by driving it up to about 3,600 r.p.m. with an auxiliary motor and by reducing the speed of the alternator until it corresponded with that of the motor. This was done by use of a neon bulb connected to the alternator and allowed to shine on the rotor of the synchronous motor. Once the motor was synchronized it could be brought up to 45,000 r.p.m. by increasing the speed of the alternator. Such a motor may be driven at much higher speeds provided the rotor laminations and the bearings can stand the strain.

FIG. 2.

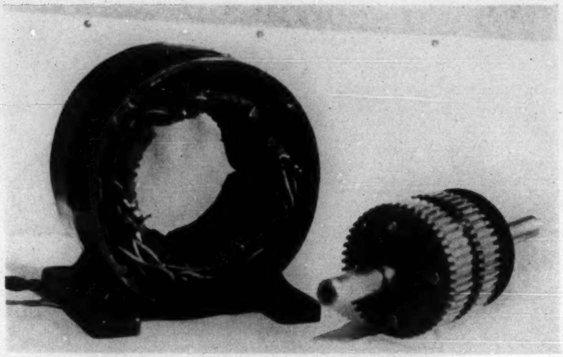


High speed induction motor showing disk with single bearing, ruptured disk, and stator coils.

It was decided that some of the auxiliary apparatus required for the synchronous motor could be done away with in an eddy current motor. A motor of this type was built as shown in Fig. 2. The movable part of this motor is a three-inch aluminum disk with a shaft supported in a single bearing. If two bearings were used the dynamic unbalance of the disk rapidly destroyed them; but with one bearing the disk acted like a top so that at 75,000 r.p.m., the highest speed attained, it ran smoothly and steadily without undue strain on the bearing. The disk ran between two circular pole pieces made of spirally wound steel laminations in which the winding is similar to the well known ring winding. There are two poles for two phase operation. The disk finally ruptured when revolving at 75,000 r.p.m. in a vacuum; the broken disk is shown in Fig. 2.

A 3,000 cycle induction alternator was constructed for driving this motor. This was made from a three horse power induction motor having forty-eight stator teeth. Figure 3 shows the construction of this machine. It was designed for two phase output at low voltage and high current. A ring of teeth in the center of the stator was milled out so that coils could be wound separately around the two remaining sets of teeth to constitute the two phases. The tops of the teeth were milled off to make the width of the slots approximately equal to the width of the teeth. The squirrel cage rotor was removed from the shaft and two stacks of blank rotor laminations separated by fiber rings were pressed on. The width of the fiber separator was made equal to the width of the slot between the two phases in the stator. Forty-eight teeth were milled in each of the rotor sections, one set being displaced from the other by ninety electrical degrees.

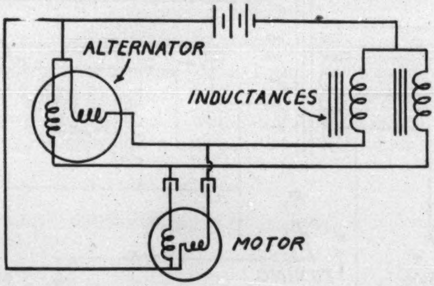
FIG. 3.



The high frequency, two phase inductor alternator used in driving the induction motor.

The circuit which proved the most satisfactory is shown in Fig. 4. With the correct values for the inductances and capacities the circuit resonated at the frequency of the alternator. This resulted in a large power output. The power output at 3,000 cycles is approximately one k.v.a.

FIG. 4.

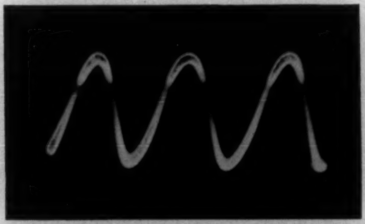


Wiring diagram of the alternator and high speed motor with resonant circuit.

Storage batteries in parallel are used to supply the direct current. The wave form of the alternator as indicated by a cathode ray oscilloscope is shown in Fig. 5.

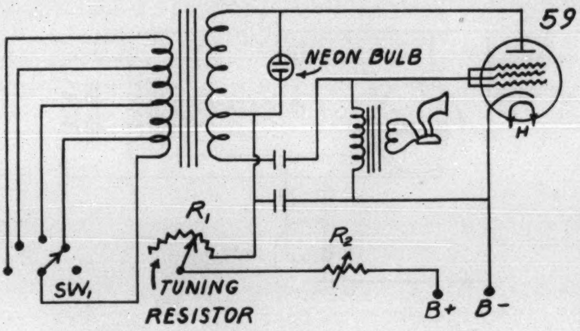
In making tests on the eddy current motor it was found desirable to have some speed measuring device which could be varied continuously over a wide range of speeds. A stroboscope was constructed consisting of a Hartley oscillator driving a neon bulb for visual observation and also driving a loud speaker unit for audible measurement of speed. The circuit is shown in Fig. 6. At first, a large variable condenser was used for varying the frequency of the oscillator but it was

FIG. 5.



Oscillograph curve showing the wave form of the high frequency alternator.

FIG. 6.



Wiring diagram of the stroboscope for measuring rotor speeds.

found that the range of frequency for any particular value of inductance was much too small and that a very large number of taps varying the inductance were required. The variable condenser was removed and a variable resistor was put in the plate supply lead for controlling the frequency. By thus varying the plate current a large frequency range was obtained for a fixed value of inductance and capacity. For instance, it is possible to vary the frequency from 20,000 cycles per minute to 130,000 cycles per minute by varying the tuning resistor from minimum to maximum resistance. Lower or higher frequencies are obtained by switching in more or less inductance in the tuned circuit.

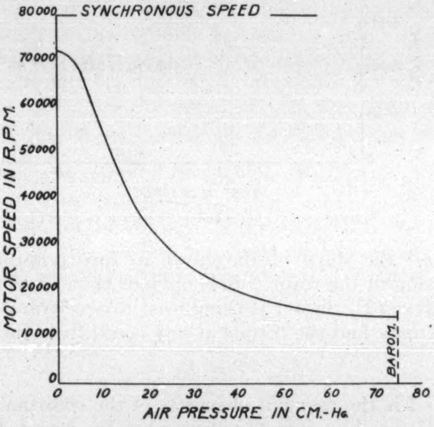
Resistor R-1 (Fig. 6) is the main tuning control and is supplied with a calibrated dial. Switch SW-1 varies the frequency in large steps by reducing the inductance of the tuned circuit. Resistor R-2 compensates for changes in "B" supply voltages. It is adjusted before each test by comparing the beat note between the loud speaker of this oscillator and a tuning fork.

The speed of the motor as well as the frequency of the alternator can be measured both visually with the neon bulb and audibly by means of the loud speaker unit. The visual method is more accurate as a spot on the motor disk appears to be standing still when its speed and the oscillator frequency are the same.

EXPERIMENTAL RESULTS.

The maximum speed that could be obtained with the motor with normal air pressure was 50,000 r.p.m. It was found that the speed rapidly increased as the air pressure was lowered showing that air resistance is very high at such speeds. The curve in Fig. 7 shows how quickly the speed increased with decrease in pressure. The imput voltage was lowered to one-quarter normal so as to accentuate the results. At normal pressure the slip was 80 per cent., but this fell to only ten per cent. with low air pressures. This ten per cent. slip was due to bearing friction. It is evident that if speeds much above these are to be obtained, the motor must be run in a vacuum.

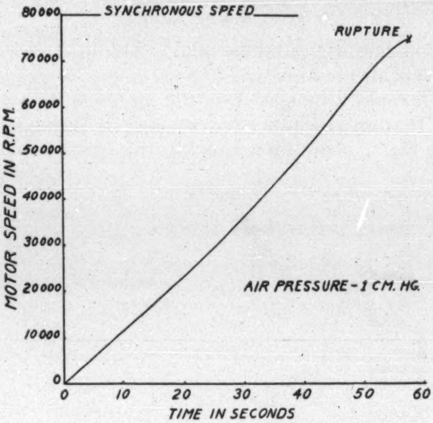
FIG. 7.



Curve showing relation between air pressure and motor speed.

The available horse power of the motor at any speed may be calculated from the curve in Fig. 8. This curve shows the time necessary for the motor to come up to any particular

Fig. 8.



Time-speed curve of the motor in a vacuum chamber.

speed and the slope of the curve at any point gives the acceleration of the rotor. It is obvious that all of the available power of the motor is being used to accelerate the rotor. Thus we may find the torque at any speed from the formula,

$$T = I\alpha, \tag{1}$$

in which I is the moment of inertia of the rotor and α is the acceleration. The horse power may be found from the formula,

$$HP = Tn/33{,}000, \tag{2}$$

in which T is the torque, and n is the number of revolutions per minute. By this method the power at 70,000 r.p.m. is found to be approximately one-half horse power.

As was mentioned before, the disk flew apart at 75,000 r.p.m. The maximum unit stress in a disk of this kind can be calculated from the equation

$$F = \frac{3+\gamma}{4}\rho\omega^2\left(b^2 + \frac{1-\gamma}{3+\gamma}a^2\right),^{1} \tag{3}$$

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[1] Timoshenko, "Theory of Elasticity," p. 69 et seq.

in which F is the maximum unit stress in the disk, γ is the Poisson ratio for the material, ω the angular velocity, b the radius of the disk, a the radius of the shaft hole and ρ the density. If there is no hole in the disk, it will stand approximately $\sqrt{2}$ times the angular velocity as shown by equation (4)

$$F = \frac{3 + \gamma}{8} \rho \omega^2 b^2.\qquad (4)$$ [2]

Using equation (3), it was found that the maximum unit stress in the eddy current motor disk at 75,000 r.p.m. was approximately 30,000 pounds per square inch. The elastic limit of the aluminum alloy is 25,000 pounds per square inch, while the ultimate strength is 35,000 pounds per square inch. At 75,000 r.p.m. the rim speed of the three inch disk was approximately 700 miles per hour. This produced high wind resistance in air.

These experiments show that it is possible to produce great speeds with electric motors in a vacuum. Their speed is limited only by the strength of the material in the disk and the ability of the bearing to withstand the strain. Commercial high speed bearings, when used as in these motors, gave no trouble even though the disk was rotating 1,250 times a second.

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[2] Ibid.

Crystal Violet as an Important Anti-Infective.—More than thirty years ago a Department of Agriculture scientist while seeking the cause of hog cholera, established the fact that it leaves the surviving animals immune, and later developed anti-hog-cholera serum. The scientist was DR. M. DORSET. He died last July, but he left as a monument to his persistent genius a new vaccine—crystal violet—which, it is believed, may be the most satisfactory product yet developed to fight hog cholera. A patent on crystal-violet vaccines—dedicated to the public welfare—has been applied for.

Dr. Dorset studied the diseases of many farm animals and developed many products for their control. But he was never fully content with his cholera serum. In recent years he and his staff sought, in particular, chemicals to mix with the deadly virus which would destroy the infective element and at the same time preserve the protective element that creates immunity to hog cholera. They experimented with various substances—phenol, glycerin, formalin and other chemicals. One of the last was crystal-violet, a water-soluble dye. Dr. Dorset's coworkers now announce that in their experiments with 200 pigs, crystal-violet vaccine has given about 99 per cent. protection against hog cholera.

C.

Aid for Those of Deficient Hearing.—The American Standards Association has announced the formation of a new committee which will establish standards of performance for audiophones and other devices to aid hearing. The committee will be headed by DR. VERN O. KNUDSEN, professor of physics and dean of the graduate school, University of California, Los Angeles.

Dr. Knudsen points out that there are no national standards by which to measure noise. A doctor can take the temperature of a patient with a thermometer calibrated according to an established standard. An eye specialist has standard lenses to measure the deficiencies of sight. So far, there is no standard meter for examining people who are hard of hearing.

Until the standard has been established and has been made available to ear specialists and instrument manufacturers, the patient will never be sure that he has the correct adjustment. Some devices work only in the higher registers; other in low registers. A standard will show the efficiency of an instrument in any register. It will determine how much an ear-aid device will amplify, how much distortion it will have, and how noisy it will be.

C.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

COLORING OF STEEL.

The coloring of steel by means of films of oxides of iron formed on the surface is a subject of frequent inquiry, the well-known temper colors formed by heating steel to a temperature several hundred degrees above room temperature being of especial interest. These colors range from light straw color to blue, this being the color most frequently desired.

This tempering treatment frequently has as its chief object the securing of suitable physical properties. Often, however, coloring is the chief object, not only for ornamentation but for securing some protection against corrosion.

The prevalent opinion is that the temper colors depend only on the temperature, blue requiring the highest temperature and yellow the lowest. Experiments at the Bureau, however, indicate that the color obtained is greatly influenced by the length of time the material is maintained at the temperature. The relationship between time and temperature is now being investigated. The time and temperature required also vary with the composition of the steel. Temper colors are due to light interference by an oxide film of varying thickness. The yellow color is due to interference and extinction of light of relatively short wave-length, and blue to extinction of light of longer wave-length, a thicker film being required.

DETERMINATION OF OXYGEN IN STEEL.

One of the most dependable methods for the determination of oxygen in steel requires the fusion of the sample in vacuo. One difficulty repeatedly encountered by investigators, is the interference of manganese, unavoidably present in the sample of steel. In previous work at the Bureau in developing the method, it was found that manganese contents as low as 0.24 per cent. caused appreciable interference. However, it appears now that with present-day apparatus, manganese interference is not nearly so serious as it was with the apparatus previously used. The employment of high-capacity pumps for the more rapid removal from the furnace chamber of the gases evolved from the melted sample, is the improvement principally responsible for reducing the amount of the manganese interference.

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* Communicated by the Director.

ARC AND SPARK SPECTRA OF COLUMBIUM.

In 1801, the English chemist, Charles Hatchett, discovered a new element in a mineral found in Massachusetts, and named it "columbium." For more than a century, this element remained a chemical curiosity, neglected by physical and technical investigators. Columbium metal of high purity was prepared by Dr. C. W. Balke in 1929 and electrodes were generously presented to the Bureau for spectroscopic studies. Recently, the metal has been given an important use in the steel industry. When alloyed in moderate and economical proportions with chromium nickel steels, it inhibits intergranular corrosion, and at the present time at least a ton a day is being refined and used for this purpose.

Since the use of spectra for chemical identification and quantitative analysis is developing rapidly, it will not be long before practical spectroscopists will need a satisfactory description of columbium spectra. No comprehensive and accurate measurements of wave-lengths have ever been published for columbium, but this deficiency will be remedied by a paper in the May number of the *Journal of Research* (RP881), which describes the conventional arc and spark spectra throughout the entire range which can be photographed with large spectrographs in air. Wave-length measurements are presented for 5,700 lines, the average probable error being less than 1 part in a million. These data should facilitate spectroscopic identification and analysis of columbium, and serve as a basis for further study of spectral structures, magnetic effects, and hyperfine structures.

SOIL CORROSION STUDIES 1934.

In 1922 the Bureau undertook to determine whether and to what extent soils were responsible for the deterioration of underground pipe. To accomplish this about 7,000 specimens of commonly used pipe materials were buried. It soon became evident that some soils were corrosive, and in an attempt to find a remedy for underground corrosion, specimens of other materials and of protective coatings were added until the total number amounted to approximately 31,500 specimens. Of these, about 22,000 have been removed and examined. The report on approximately 2,000 ferrous specimens removed in 1934 will be published as RP883 in the May number of the *Journal of Research*. The results given in this and earlier reports can be summarized as follows:

Soil conditions even within a very limited area are so varied that the result of a single or a very few observations of corrosion is of little value. In order to determine the corrosiveness of a soil or the life of a pipe line a large number of observations must be made. The value of deepest pit (average of several observations) on an area of pipe surface exposed to a single soil condition will increase if the area is increased, i.e., one can expect to find a deeper pit on 100 feet of pipe than on 10 feet of pipe in the same soil. Pitting data are not comparable unless the observations are for equal areas exposed for the same time to the same soil. In most soils the rate of corrosion decreases with time but for some soils the penetration is nearly proportional to the period of exposure.

The average value of the maximum pit after 12 years of soil action is slightly less for wrought iron than for the other commonly used pipe materials, but the standard error of the averages indicates that the apparent differences in the materials may be accidental or within the limits of errors of observation. The average values of penetration for both pure open-hearth iron and for open-hearth steel to which 0.2 per cent. of copper has been added are rather definitely higher than for the more commonly used pipe materials. In some soils, especially those containing so-called alkali, cast iron corrodes somewhat more rapidly than steel, but the corrosion products appear to have more strength as pipe material. The several varieties of cast iron, with the exception of one containing 15 per cent. of nickel, corrode at approximately the same rates.

The addition of a small percentage of chromium to steel reduces the loss of weight but has little effect on the maximum pit depth. Indeed, in soils containing chlorides the corrosion may be concentrated and accelerated. Very limited data suggest that the addition of nickel with or without chromium tends to reduce both loss of weight and pit depths.

EFFECT OF MAGNESIA ON PORTLAND CEMENT.

Excessive amounts of magnesia are known to affect adversely the strength, soundness, and volume changes of portland cement. There are indications that the intensity of these changes is related to the state of the magnesia in the clinker. As a preliminary step in the quantitative determination of this relation studies were made of the boundaries of primary phase volumes in the quaternary system lime-magnesia-dicalcium silicate-pentacalcium-trialuminate. It was found that the magnesia as primary phase occupies a relatively large volume. In some compositions magnesia becomes the primary phase in concentrations as low as 5 per cent., and in all compositions it becomes the primary phase in concentrations of more than 33 per cent. A complete account of this work will be published as RP884 in the *Journal of Research* for May. A diagram will be included showing the primary phase relations of magnesia in the quaternary system.

INFLUENCE OF PARTIAL PREHYDRATION AND CURING TEMPERATURES ON CEMENT AND CONCRETE.

The possibility of producing cements having low heats of hydration by treating standard and high-early-strength portland cements with steam at atmospheric pressure was recently investigated at the Bureau. Two different degrees of hydration were studied, the ignition loss of each cement being increased to approximately 3 and to 5 per cent., respectively. The effects of these partial hydrations of the cements on the heats of hydration and on the compressive strengths of concretes made of the cements were determined for the ages of 7, 28, 90 days, and 1 year for different curing temperatures.

The resistance of mortar bars, made of the untreated and of the prehydrated cements, to the disintegrating action of sodium sulfate solutions was studied by determining the expansion of mortar bars immersed in these solutions. It was found that the 3-per cent. prehydration of the standard portland cements caused a reduction in the average heat of hydration at 7 days of approximately 13 per cent., but at 1 year this reduction amounted to only about 6 per cent. of the heat of hydration of the untreated cements. The 5-per cent. prehydration of these cements caused a reduction in the average heat of hydration at 7 days of approximately 30 per cent., and at 1 year a reduction of about 20 per cent. The prehydrations of the high-early-strength cements caused only about two-thirds the decrease in the average heat of hydration that was found (for the same degrees of prehydration) for the standard portland cements.

The heats of hydration of the 5-per cent. prehydrates of the standard portland cements all fell within the upper limits specified for low-heat cements used in the construction of Boulder Dam.

The compressive strengths of concretes made of the prehydrated cements were all lower than the compressive strengths of the concretes made of untreated cements. This reduction was greatest at 7 days and decreased with age. The 5-per cent. prehydration reduced the strength to a greater extent than the 3-per cent. at all ages. The standard and high-early-strength cements cured at 70° F. had greater strengths up to 90 days than the Boulder Dam cements, but after that age the increase in strength of the Boulder Dam cements exceeded that of the other two types. At 150° F. curing, the Boulder Dam cements had strengths of the same order as the untreated portland at all ages.

The expansion measurements showed that, with the exception of cements with high tricalcium aluminate content, the prehydrations increased the resistance of the cements to the disintegrating action of sodium sulfate solutions. The 5-per cent. prehydrates had greater resistance to the sulfate action than the 3-per cent.

For a complete account of this work, RP887 in the May number of the *Journal of Research* should be consulted.

ACCELERATED WEATHERING TEST FOR ASPHALTS AND OTHER MATERIALS.

Actual outdoor exposure furnishes a satisfactory quality test for materials that deteriorate rapidly, but, obviously, this method of test is not applicable to materials which are intended to withstand long periods of exposure with but little alteration.

An accelerated weathering test, in which samples are exposed alternately to light and heat from a carbon arc lamp, a period of refrigeration, and a period during which water is forcefully sprayed on the surfaces that have been exposed to the light, has been in use at the Bureau for several years in studying the weathering characteristics of such materials, and is being used by a number of testing laboratories and manufacturers.

Experiments at the Bureau have shown that the weathering process gradually transforms bituminous materials, cotton, wool, rubber, and similar materials into products that are soluble in water. A modified accelerated weathering test has, therefore, been developed, in which immersion in distilled water is substituted for the water spray. This method permits the products of weathering to be collected for further examination. A more complete description of the apparatus and procedure will be found in the May number of the *Journal of Research* (RP886).

WEATHERING QUALITY OF ROOFING FELTS.

Owing to the increasing scarcity of suitable rags, the need for substitute materials is a problem confronting the manufacturer of paper felts and prepared roofings. In order to determine the effect of different fibers on the life and serviceability of asphaltic roofing materials, an investigation was made at the Bureau by the Research Associate of the manufacturing and industrial research committee of the Asphalt Shingle and Roofing Institute. Experimental felts composed of cotton rags, old jute and manila bagging, old newspapers, sulphite pulp, and wood sawdust were made for the investigation and exposed outdoors to determine their behavior to weathering. Inspection and tests showed that there was no significant difference in the resistance to weathering of asphalt roofing which may be attributed to the kind of fiber or combination of fibers employed. A more complete report of this work will be published as RP888 in the May number of the *Journal of Research*.

ALKALI DEGRADATION OF WOOL PROTEIN.

Previous work has shown that the disulfide group constitutes one of the most important connecting links in the wool molecule. The disulfide group is particularly susceptible to attack by alkalies and, as a result, wool is readily degraded by alkaline reagents. In addition to its importance in practical wool processing, the problem of labile sulfur in biological materials has recently become one of the most important in biochemical research.

Labile sulfur is an arbitrary name given to that portion of the sulfur which splits off from sulfur-containing substances on treatment with various alkalies or with alkaline solutions of various reagents such as lead acetate, lead nitrate, etc. It is obvious that the value obtained for the amount of labile sulfur will depend on the method used and the specific conditions of treatment. There have been postulated several mechanisms, which will be discussed later, for the splitting of sulfur from cysteine, cystine, and other mercaptans and disulfides. In the case of proteins, however, results which have appeared to be anomalous have led a number of investigators to assume the existence of a number of sulfur combinations in proteins.

RP885 in the May number of the *Journal of Research*, describes the behavior of sulfur in wool under conditions which are very different from those ordinarily used in studies on sulfur lability, and points out a probable mechanism by which sulfur is split from wool during alkali treatment. The data indicate that the primary process in the alkali cleavage of the disulfide linkage consists in a hydrolytic rupture of the disulfide group with the formation of a sulfydryl compound and a sulfenic acid. The sulfenic acid is extremely reactive and unstable in alkaline solution and immediately loses hydrogen sulfide and forms an aldehyde. The results of the investigation indicate that the existence of labile sulfur in proteins is not an indication that the bulk of the sulfur is present in more than one form.

PAPER DISTORTION IN MULTICOLOR OFFSET PRINTING.

The following is an abstract of a paper to be presented by C. G. Weber of the Bureau's paper section at the meeting of the Graphic Arts Research Bureau, in Washington on May 11.

The distortion of paper in multicolor offset printing with the attending difficulty of obtaining register of successive color prints has been responsible for many serious difficulties. It appears also that, of the problems involving paper, distortion has been least clearly understood. The requirements of the offset process are exacting, in that the dimesnions of paper must be held constant during printing. Practically all dimensional changes affecting register are caused by moisture content changes, and since paper is a highly hygroscopic substance with high coefficient of linear expansion per unit of moisture content change, constant moisture content must be maintained to avoid distortion. Results of the experimental work at the Bureau indicate that constant moisture content can be maintained by proper hygrometric treatment. However, best results require not only air conditioning in the press room, but complete control of the moisture content from the paper machine to the printing press, plus special treatment to compensate for the effects of press moisture.

To meet all these requirements is difficult; however, two methods of obtaining the desired results have been found. Papers conditioned first to a very high moisture content, then to equilibrium with the pressroom atmosphere by desorption and printed under constant atmospheric conditions showed no distortion. Also, papers prepared for printing by conditioning to equilibrium with relative humidity 5 to 8 per cent. above the pressroom condition gave satisfactory results. The former condition will be obtained if the paper is never dried below hygrometric equilibrium with the pressroom condition. However, since most papers are dried to a moisture content of less than 5 per cent. in manufacture to permit calendering, this procedure is not feasible for general practice. The second method of treatment is more promising for most plants.

Where facilities are not available for conditioning paper to relative humidity 5 to 8 per cent. above the pressroom humidity, the moisture content should be adjusted at the mill, and the paper protected during transit and storage so that it will reach the printing plant with a moisture content 0.5 to 1.0 per cent. above equilibrium with humidity maintained in the pressroom. In no instance should paper be shipped with moisture content below equilibrium with the relative humidity of the pressroom it is to be used in, and it should always be adequately protected against changes of moisture content by water-resistant wrappers. If this procedure is followed, printing plant conditioning will be greatly simplified, and wavy edges will be avoided.

CONGRESSIONAL ACTION IN REFERENCE TO WEIGHTS AND MEASURES.

Miscellaneous Publication M122, which was released last month, presents a detailed account of the steps taken to secure uniformity in weights and measures in the United States. It is an historical account of the various plans and proposals made in or to Congress on this subject, based upon actual records of Congress and other original documents, and culminating in the passage of the Joint Resolutions of Congress of 1836 and 1838.

The most important plans submitted on this subject were those of Thomas Jefferson and John Quincy Adams. Neither plan was adopted, but the reports of Jefferson and Adams have great historical and technical value for students in this field. The first act of Congress relative to establishing uniformity in weights and measures was the Surveyor Act of 1799, but since no standards had been adopted, this legislation was not put into effect at the time of its passage.

The first act of Congress to be put into effect promptly after passage was the Mint Act of 1828 adopting the troy pound and providing for uniformity in the weight of the coins of the United States. After the passage of this act, the work of securing uniformity in weights and measures was seriously begun. This work finally resulted in the construction and adoption of standards as provided in the Joint Resolutions of 1836 and 1838.

With the completion and delivery of the standards of weight and measure as provided for in these joint resolutions, there was at last established a ground work upon which a weights and measures structure, uniform throughout the country, could be erected. The legal basis having been laid down by these resolutions and by the Mint Act of 1828, it was then that the States began to proceed independently, but at the same time in reasonable harmony, to effect a practical control of this fundamental of Government—uniformity in the standards of weight and measure.

Copies of this publication are obtainable from the Superintendent of Documents, Government Printing Office, Washington, D. C., at 5 cents each.

THE FRANKLIN INSTITUTE.

STATED MONTHLY MEETING, MAY 20, 1936.

The stated monthly meeting of the Institute was called to order at 3:30 o'clock in the afternoon, by the President of the Institute, Mr. Nathan Hayward. The President stated that the minutes of the April meeting had been printed in full in the May issue of the Journal of the Institute, and that if there were no objections, the minutes would stand approved as printed. No objection was offered and the minutes were declared approved.

He then announced that the next business was the presentation of the medals which had been recommended by the Committee on Science and the Arts, and the award of certificates of honorary membership. The order of the awards proceeded as follows:

Presentation of Longstreth Medals to	*Sponsor*
Alfred V. deForest, President Magnaflux Corporation New York City, New York and	Mr. Clifford W. Bates
William E. Hoke Consulting Engineer Baltimore, Maryland	Mr. Clifford W. Bates
Peter P-G. Hall, President The Hall Planetary Company Philadelphia, Pennsylvania	Mr. Warren P. Valentine
Elmer A. Sperry, Jr., Vice-President Sperry Products, Inc. Brooklyn, New York	Mr. W. Laurence LePage
Presentation of Wetherill Medal to	
Albert L. Marsh, President Hoskins Manufacturing Company Detroit, Michigan	Mr. Edward L. Forstall
Presentation of Clark Medal to	
Joseph Becker, D.Eng., President The Koppers Construction Company Pittsburgh, Pennsylvania	Mr. Owen Brooke Evans
Presentation of Levy Medal to	
Mayo D. Hersey, A.M. Brown University Providence, Rhode Island	Dr. Thomas D. Cope
Presentation of Potts Medal to	
Felix A. Vening Meinesz, Ph.D. University of Utrecht Utrecht, Holland	Dr. Charles B. Bazzoni

Presentation of Cresson Medals to
GEORGE O. CURME, JR., Ph.D., Vice-President — Mr. Walter H. Fulweiler
Carbide and Carbon Chemicals Corporation
New York City, New York
ROBERT J. VAN DE GRAAFF, Ph.D. — Mr. Walter C. Wagner
Massachusetts Institute of Technology
Cambridge, Massachusetts

Presentation of the Franklin Medal and Certificate of Honorary Membership to
FRANK BALDWIN JEWETT, Ph.D., D.Sc., D.Eng.,
LL.D., President and Director — Dr. Frederic Palmer, Jr.
Bell Telephone Laboratories, Inc.
New York City, New York

Presentation of the Franklin Medal and Certificate of Honorary Membership to
CHARLES FRANKLIN KETTERING, D.Eng., D.Sc. — Dr. Frederic Palmer, Jr.
Director, General Motors Research Laboratories
General Motors Corporation
Detroit, Michigan

Music, Piano Selections.
Dedication . . . . *Schumann*
Transcribed by Liszt
Londonderry Air . . . . *Percy Grainger*

MR. GUY MARRINER, Associate Director in charge of Music Section

Papers.

"Idea to Industry"
by
DR. C. F. KETTERING

"Telephony—Child of Organized Scientific Research"
by
DR. F. B. JEWETT

Adjourned.

HENRY BUTLER ALLEN,
Secretary.

(An extended account of the proceedings of Medal Day will be published in an early issue of the JOURNAL.)

COMMITTEE ON SCIENCE AND THE ARTS.

(*Abstract of Proceedings of Stated Meeting held Wednesday, May 13, 1936.*)

HALL OF THE COMMITTEE,
PHILADELPHIA, MAY 13, 1936.

MR. JOHN A. VOGLESON *in the Chair.*

The following reports were presented for first reading:

No. 3011: Mechanical Grate for Gas Generators.
No. 3016: "Transitorq."

GEO. A. HOADLEY,
Secretary to Committee.

LIBRARY NOTES.

RECENT ADDITIONS.

Adams, Orville. Elements of Diesel Engineering. 1936.

American Gas Association. Proceedings. Seventeenth Annual Convention. 1935.

American Institute of Chemical Engineers. Transactions 1934–35, Volume 31. 1936.

American Medical Association. New and Nonofficial Remedies. 1935.

American Pharmaceutical Association. The National Formulary. Sixth Edition. Official from June 1, 1936. 1935.

American Society of Mechanical Engineers. Rules for Construction of Power Boilers and Other Pressure Vessels and for their Care in Service Known as the A.S.M.E. Boiler Construction Code. 1935.

Barclay, Hartley W. Ford Production Methods. 1936.

Bennett, H. The Chemical Formulary. A Condensed Collection of Valuable, Timely, Practical Formulæ for Making Thousands of Products in all Fields of Industry. Volume 3. 1936.

Boylston, H. M. An Introduction to the Metallurgy of Iron and Steel. Second Edition. 1936.

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Dixon, S. M., and H. J. Grose. Deterioration of Structures of Timber, Metal, and Concrete Exposed to the Action of Sea-Water. Fifteenth Report of the Committee of the Institution of Civil Engineers. Published by the Great Britain, Department of Scientific and Industrial Research. 1935.

Durand, William Frederick. Aerodynamic Theory. A General Review of Progress under a Grant of the Guggenheim Fund for the Promotion of Aeronautics. Volume 6, 1936.

Eucken, A., and K. L. Wolf. Hand-und Jahrbuch der chemischen Physik. Bd. 9. Abschnitt 3–4: Anregung der Spektren, von W. Hanle; und Molekülspektren von Lösungen und Flüssigkeiten, von G. Scheibe und W. Frömel. 1936.

Golding, E. W. Electrical Measurements and Measuring Instruments. Second Edition. 1935.

Haynes, Williams. Men, Money and Molecules. 1936.

Heisel, Carl Theodore. Mathematical and Geometrical Demonstrations. Second Edition. 1934.

Heitler, W. The Quantum Theory of Radiation. 1936. International Series of Monographs on Physics.

Hildebrand, Joel H. Solubility of Non-Electrolytes. Second Edition. 1936. American Chemical Society. Monograph Series No. 17.

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Jane's All the World's Aircraft. 1935.

Krieger, Louis C. C. The Mushroom Handbook. 1936.

Kunitz, Han A. Oil Heating Handbook. 1936.

Lucas, Howard J. Organic Chemistry. 1935.

Mackey, Charles O. Graphical Solutions. 1936.

MACMILLAN, WILLIAM DUNCAN. Dynamics of Rigid Bodies. First Edition. 1936.

MERSEREAU, SAMUEL FOSTER. Materials of Industry their Distribution and Production. Revised Edition. 1936.

MILLARD, E. B. Physical Chemistry for Colleges. A Course of Instruction Based upon the Fundamental Laws of Chemistry. Fourth Edition. 1936. International Chemical Series.

MINERVA. Jahrbuch der gelehrten Welt. Zweiunddreissigster Jahrgang. Abteilung Universitäten und Fachhochschulen. 1936.

MOORE, HERBERT F. Textbook of the Materials of Engineering. Fifth Edition. 1936.

POUND, ARTHUR. Industrial America Its Way of Work and Thought. 1936.

RAMSEY, CHARLES GEORGE, AND HAROLD REEVE SLEEPER. Architectural Graphic Standards for Architects, Engineers, Decorators, Builders and Draftsmen. Second Edition. 1936.

ROTHERY, WILLIAM HUME-. Structure of Metals and Alloys. 1936. Monograph and Report Series No. 1.

SAMUELS, WILLIAM. Engine Bearing Loading. 1935.

SCHLESINGER, G. Die Werkzeugmaschinen: Grundlagen, Berechnung und Konstruktion. I: Textband. II: Tafelband. 1936.

SEELY, FRED B., AND NEWTON E. ENSIGN. Analytical Mechanics for Engineers. Second Edition, Rewritten. 1933.

SKIRL, WERNER. Elektrische Messungen. 1936. Siemans Handbuch VI Band.

SMITH, ROBERT H. Text-Book of Advanced Machine Work. Eleventh Edition, Revised and Enlarged. 1930.

SOUTHWELL, R. V. An Introduction to the Theory of Elasticity for Engineers and Physicists. 1936. The Oxford Engineering Science Series.

TSCHIRCH, A., AND ERICH STOCK. Die Harze. Die botanischen und chemischen Grundlagen unserer Kenntnisse über die Bildung, die Entwicklung und die Zusammensetzung der pflanzlichen Exkrete. Dritte umgearbeitete Auflage von A. Tschirch. Die Harze und die Harzbehälter. II Band 2 Hälfte 2 Teil. 1936.

TWENEY, C. F., AND I. P. SHIRSHAV. Hutchinson's Technical and Scientific Encyclopædia. In Four Volumes. No date.

WILLIAMS, GATENBY. William Guggenheim. 1934.

BOOK REVIEWS.

STARS AND TELESCOPES, by James Stokley, preface by Dr. Walter S. Adams. 319 pages, plates, 15 × 22.5 cms. New York and London, Harper & Brothers, 1936. Price $3.00.

It is true that appreciation often is necessary to envision beauty in various objects. This is dependent upon background and education, and is regardless of the spectacular nature of the thing or things thought beautiful. Thus, a sanitation specialist may derive much pleasure in examining a modern sewage disposal plant, or a chemist may appreciate and enjoy a visit to a petroleum refinery, while to the great majority of people, even though knowing that such activities of man are necessary, the very nearness of such operations is repulsive. Appreciation and the ability to see beauty in much of man's activity on earth is therefore limited to a very few. There is much to be done to encourage appreciation along these lines.

Nature's work is spectacular to a larger number of people, because much of it is beyond comprehension and is available for all to see. To man, it first offers beauty, then a challenge to inquiry. Standing in an open field in the country on a very clear night, the average person can not help but be impressed with the beautiful decorations in the sky. An excellent illustration of the completeness of the impression of such beauty on a cross section of humanity can be had by observing the audience in a planetarium such as the Fels Planetarium in Philadelphia. At the beginning of the performance the lights are dimmed, and then at the proper time, the heavens are suddenly illuminated with countless bright stars. The audience is spellbound; breathless at the beautiful sight. For several moments there is not a stir, and at the conclusion of the program, by the expressions of the audience, it is not difficult to see that greater appreciation has been established.

Mr. Stokley in his book tells us that the heavenly beauties have been observed in awe by humans as long as there are historic records. He tells us that the ancients created interesting legends about the various stars and diagrams in the sky, such as the Great Dipper, known also as the Great Bear. With this particular diagram it is interesting to note that these stars have been called a bear in widely scattered parts of the earth, among peoples who can scarcely have been in communication—the Greeks, the Finns, the people of the Tigris-Euphrates, and the American Indians.

As human knowledge progressed theories were evolved and proved and disproved with relation to the heavenly bodies and the Earth's connection thereto. The motions of the Earth and other bodies was established and the conception that the Earth is round made possible. This book reveals that though many people credit Columbus with first suggesting that the Earth is round, this idea was generally accepted among scholars many centuries earlier. Pythagoras, who died about 500 B.C., and his followers, taught that the Earth was a sphere, suspended with support.

Mr. Stokley's presentation of the history of man's knowledge of the universe is facinating and inspiring. He reviews how scientific instruments were devised and their applications. A very clear description of different telescopes from the earliest to the modern, how they were made and work, is given. There is an interesting insight on the practical application of knowledge of the heavens, such as the establishment of a standard of time and the determination of locations on the Earth, as is made by navigators. There is fully explained the methods used in finding distances of stars, their characteristics, such as temperatures on their surfaces, atmospheres around them and their composition. Each of the planets is taken up and discussed and there is laid out for the reader interesting speculations and theories of modern thought. In fact the reader is brought right up to date in this; the treatment of the present limitations of knowledge is particularly noteworthy.

The last few pages of the book are devoted to a chapter on "Life in the Universe" a fitting conclusion to an admirable work. After finishing the book with this, few readers will fail to be inspired with zest for more knowledge of this fascinating subject.

The book contains a glossary, the value of which cannot be over-estimated, especially to those readers not familiar with and not wishing to take the time to recall the meaning of the various terms and names. A subject index of unusual clarity makes for usefulness as reference. No special prerequisites are necessary to get the fullest enjoyment from this work. Mr. Stokley, an unquestioned authority, has an unsurpassed knack of presentation in everyday language of a most interesting science. The result is a book that broadens knowledge through an easy path, thereby creating a better appreciation of the beauties of the heavens, and encouraging the search for truth.

R. H. OPPERMANN.

FUN WITH ELECTRICITY, by A. Frederick Collins, F.R.A.S. 238 pages, illustrations, 13 × 19.5 cms. New York and London, D. Appleton-Century Co., Inc., 1936. Price $2.00.

This book is a "find" for the amateur experimenter and hobbyist who desires to obtain enjoyment by constructing the apparatus and observing the mystifying phenomena of electricity. It is divided into three parts, direct currents, alternating currents and high frequency currents. Careful description is given for each kind of apparatus to be built and each may be constructed very cheaply. Some of the spectacular experiments explained are: an incandescent lamp which is submerged in a glass of water and which lights up to brilliancy without any connecting wires; an aluminum ring that is thrown several feet into the air by a magnet instead of being attracted to it; a metal disc, some mercury in a glass, and a metal ball, any one of which will rotate when it is placed over the pole of a magnet; boiling water and frying an egg by the heat of electromagnetic induction etc. The author suggests that the performance of these experiments may provide much enjoyment for an audience.

Aside from the purely enjoyable aspects of doing these things the experimenter will gain a valuable knowledge of electricity and magnetism.

R. H. OPPERMANN.

MATHEMATICS OF MODERN ENGINEERING, Volume 1, by Robert E. Doherty and Ernest G. Keller. 314 pages, illustrations, tables, 15 × 23.5 cms. New York, John Wiley & Sons, Inc. London: Chapman & Hall, Ltd., 1936. Price $3.50.

With the progress of the sciences and engineering in recent years, the use of mathematics has greatly increased. This has served to emphasize the mathematical bond that exists between the sciences. In engineering, mathematics is an essential adjunct to the scientific method: in clarifying thinking, correlating and interpreting data, aiding discovery, setting forth analogies, and expressing scientific results in a usable form. Necessarily then, in the educational process, mathematics must be given special place and special attention. To those undergraduate students in colleges who are interested in the higher levels of engineering service, this book is intended especially, although graduates may and, no doubt, often will find it of much value particularly from the viewpoint of how to use mathematics in engineering.

The book is the first volume of a two volume set. The only knowledge presupposed for understanding it is the calculus. There are four chapters: mathematical formulation of engineering problems; basic engineering mathematics; vector analysis; Heaviside's operational calculus. The structure of the book is such that its approach is from the engineering side, rather than the mathematician's—how to reduce the phenomena under observation to mathematical equations, and then how to solve them. The former is something that a large number of practicing engineers have great difficulty in accomplishing, and is one of the greatest needs. The scope of the book includes practical engineering work dealing with electrical and mechanical problems. Throughout the text exercises illustrate a great variety and their answers appear in the back. One of the most noteworthy features are the summaries of the various processes and methods treated upon, appearing where necessary at the end of sections of the book. These summaries are very similar to notes of an abstract nature that all students are inclined to make while studying. Their value, therefore, can readily be appreciated. A bibliography is appended, classified by subject, giving references to additional and related material of the text.

The engineer desiring to study a subject that will aid him greatly in the solution of his problems will do well to peruse this work.

R. H. Oppermann.

The Structure of Metals and Alloys, by William Hume-Rothery, M.A., D.Sc. 120 pages, illustrations, 14 × 22 cms. London, Institute of Metals, 1936. Price 3s.6d.

This 120 page monograph is the first of a series to be made by the Institute of Metals. The author needs no introduction to those familiar with the subject and his work here presented, to say the least, is unusual and noteworthy. It is a work which summarizes certain selected subjects under this heading which throw light on the fundamental principles. These include the electronic background of metallurgy, the crystal structure of the elements, atomic radii of the elements, primary metallic solid solutions, intermediate phases in alloy systems, and imperfections in crystals.

Necessarily the book is illustrated appropriately, but on reading through one is impressed with the masterly coördination of the text with the illustrations, their arrangement and structure. For instance, in the chapter on the crystal structure of the elements the diagrams and descriptive matter are so interwoven with consequent clarity that the reader receives an impression not easily forgotten. References indicated throughout the text and noted at the end of each chapter constitute a good bibliography of each subject treated.

Other features that should make for wide circulation of the book are: no higher mathematics is contained in it, no use is made of crystallographic notations, wave mechanical theory is merely mentioned and passed by, and thermodynamics are not even referred to.

The book can benefit a large number of scientific men, of widely diversified interests.

R. H. Oppermann.

Solubility of Non-Electrolytes, by Joel H. Hildebrand, Ph.D., Professor of Chemistry, University of California. 203 pages, illustrations, 15.5 × 23 cm. American Chemical Society Monograph No. 17, 2d Edition. New York, Reinhold Publishing Corp., 1936. Price $4.50.

Since "Solubility" was published in 1924 theoretical and experimental contributions of Debye and others have so advanced our knowledge of electrolytes that this division of the subject is now dropped for reasons of expediency and the restricted "of Non-Electrolytes" added to the title. It will be regrettable if this disseminates the notion of inherently different fundamentals. The approach to the subject represented by this volume is best fitted to teach the opposite. Eventually "Solubility" should be brought under one roof, however large the structure be.

The book has been otherwise minutely revised and rewritten, particularly the material on polarity and internal pressures, in keeping with the much advanced maturity of these subjects. Likewise the sections on vapor pressures of liquid mixtures and metallic solutions show great metamorphosis. The number of chapters has been cut from eighteen to thirteen, mainly by consolidating the several chapters on ideal solutions and Raoult's law.

The thermodynamic treatment remains essentially unchanged, following Lewis and Randall in the main. One notable exception is the use of each pure component as its standard state instead of following the practice for dilute electrolytes of taking the pure liquid as its standard state and for the solute its infinitely dilute solution. This results in symmetry of expression for solvent and solute and facilitates comparisons between solutes in different solvents. The treatment of the solubility of solids is somewhat abbreviated and different forms of the equations used. The three dimensional figure for relations between vapor pressure, temperature and solubility of a solid is a great improvement over its earlier counterpart.

An obvious misprint occurs in 1.16, p. 188. There have been numerous changes in detail, such as values of constants, additions of titles to tables, expansion and contraction in various mathematical sections. Important deductions can be picked up from black face type (formerly in the less prominent italics), adding greatly to the practical value of the book. Years of study and use will not exhaust the implications of its method. The reading is not easy but it has been improved by a new conciseness of style in many sections rewritten, leading to condensations of as much as one-third without loss of substance.

"Solubility of Non-Electrolytes" takes a second considerable step in the direction in which theory and fact may be eventually reconciled. It presents the most comprehensive view of the field extant, resting on sound thermodynamics and tested by experimental fact. By no means are the results in full agreement but divergences are being revealed and evaluated in measurable degree by advances in knowledge of intramolecular structure and intermolecular forces.

LESLIE R. BACON.

AN INTRODUCTION TO THE METALLURGY OF IRON AND STEEL, by H. M. Boylston, Second Edition, 563 pages, illustrations, 15 × 24 cms. New York, John Wiley and Sons, Inc. Price $5.00.

This is the second edition of a book that introduces to the reader the basic knowledge of the metallurgy of iron and steel, a book that can be very profitably employed in reading assignments. Its make up is such that it affords interesting reading, its construction presents treatment of the various subjects in concise form. The author is gifted in having the ability to hold the attention of the reader particularly over sections of the book descriptive of apparatus with which the reader may not be familiar.

He begins with a historical and economic background of the iron and steel industry and then proceeds in order through chemical and physical principles, the raw materials of the iron industry, the manufacture of pig iron, the foundry, and wrought iron. Following this he treats on the steel making processes, the Bessemer process, the open-hearth process and the electric furnace process. The manufacture of commercially pure iron is then taken up and special emphasis is placed on the chapter on ingots and ingot making. The reader who is mechanically inclined should find the part devoted to the mechanical treatment of steel very worth while. It is divided into hot work and cold work. An introduction is made to an important branch of the science, namely, the metallography and heat treatment of iron and steel.

The style of writing, the illustrations and drawings, make the book most impressive. Because of this it should be pointed out that the book is not a cold, matter of fact, text. Rather is it an admirable exposition of technical processes in a manner easy to grasp, even catching to a lay reader.

Workers in the iron and steel industry therefore can profit to a large extent by reading it. The book also has great educational value to every engineer and technical man whether interested primarily in this industry or not.

R. H. OPPERMANN.

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS.

Report No. 550, Cooling Characteristics of a 2-Row Radial Engine, by Oscar W. Schey & Vern G. Rollin, 10 pages, illustrations, diagrams, 23 × 29 cms. Washington, Superintendent of Documents, 1936. Price ten cents.

Cooling tests were conducted on a calibrated GR-1535 Pratt & Whitney Wasp, Jr., engine installed in a Vought XO4U-2 airplane. The tests were made in the N.A.C.A. full-scale tunnel at air speeds from 60 to 120 miles per hour, at engine speeds from 1,500 to 2,600 r.p.m., and at manifold pressures from 19 to 33 inches of mercury absolute. A Smith controllable propeller was used to facilitate obtaining the different combinations of engine speed, power, and manifold pressure.

The results of the tests showed that an air speed of 120 miles per hour was necessary for the satisfactory cooling of this engine, as installed in a Vought XO4U-2 airplane with air of standard sea-level density when operating at full throttle at an engine speed of 2,500 r.p.m. Increasing the brake horsepower 50 per cent. resulted in an increase of 13 to 20 per cent. in the temperature difference between the air and the cooling surface. When the air speed was increased from 60 to 120 miles per hour, there was a decrease in the temperature difference between the cooling air and the cooling surface of only 17 per cent. The same percentage change in temperature difference between the cooling air and the cooling surface was obtained for a given change in power when the manifold pressure was varied as when the engine speed was varied. The effect of the attitude of the airplane on the cylinder temperatures was small; the temperatures increased or decreased slightly, depending on the location of the cylinder. The difference in cylinder temperatures obtained with 3-blade and 2-blade propellers was negligible. The heat loss through the oil radiator was equal to from 3 to 6 per cent. of the heat going into indicated power, depending on engine speed and mixture strength.

Report No. 551, Aircraft Compass Characteristics, by John B. Peterson & Clyde W. Smith, 11 pages, tables, 23 × 29 cms. Washington, Superintendent of Documents, 1936. Price ten cents.

A description of the test methods used at the National Bureau of Standards for determining the characteristics of aircraft compasses is given. The methods described are particularly applicable to compasses in which mineral oil is used as the damping liquid. Data on the viscosity and density of certain mineral oils used in United States Navy aircraft compasses are presented. Characteristics of Navy aircraft compasses IV to IX and some other compasses are shown for the range of temperatures experienced in flight.

Results of flight tests are presented. These results indicate that the characteristic most desired in a steering compass is a short period and, in a checking compass, a low overswing.

R.

CHEMICAL DISCOVERY AND INVENTION IN THE TWENTIETH CENTURY, by Sir William A. Tilden. Revised by S. Glasstone. 492 pages, plates, 14 × 21 cms. New York, E. P. Dutton & Company, 1936. Price $4.00.

A cultural book with the purpose of informing the public on the developments of chemistry which are having more and more to do with our daily lives. It is claimed that more knowledge by the layman of just what the word chemistry constitutes would not only add to his general education but oftentimes would save considerable time, money, and speculation in a business way.

The book opens with an introductory chapter describing the various parts that chemistry plays in industry, chemical research in England—how it was stimulated and helped, chemical education in general, the training of the research worker, in fact sufficient to convey a general idea of the position of chemistry as the basis of a calling or profession.

Following this, the book is divided into four parts. The first section deals with laboratory equipment and special apparatus of recent invention, vacuum pumps, electric furnaces, and chemical devices used in trade processes, such as dyeing and brewing. The second part deals with principles of chemistry, electric discharge in gases, discovery and properties of radium, the structure and disintegration of atoms, electrolysis, and catalysis and catalysts etc. The third part is on modern applications of chemistry, such as processes for obtaining oxygen and nitrogen from the air, the making of incandescent gas mantles, modern drugs, synthetic perfumes and essential oils, cellulose products and explosives, artificial nitrates etc. The fourth part gives a review of the concurrent progress in organic chemistry and of the discoveries which have been made with regard to saccharine, proteins, hydrocarbons, enzymes, and ferments.

The book at hand is the sixth edition within twenty years. Its popularity therefore can hardly be questioned. It is profusely illustrated with photographs and drawings and there is an extensive subject index. While the book was originally intended for non-technical readers there is a wealth of educational information that will appeal more especially to those who have some knowledge of science also. The book is not a text in the usual sense, and in this respect the ability of the author is noteworthy, in describing chemical developments and processes in non-technical language so that it is readily understandable. To all those with a fair background of education, the book can be recommended as worthwhile.

R. H. OPPERMANN.

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The Romance of Chemistry, by William Foster, 497 pages, plates, 14.5 × 22 cms. New York and London, D. Appleton-Century Company, 1936. Price $3.00.

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CURRENT TOPICS.

China Clipper Pioneers Refrigeration.—M. BREITUNG (*Refrigerating Engineering*, Vol. 31, No. 2). Refrigeration played a part in the carefully planned flight of the giant China Clipper to Manila, initiating a new air route to the Orient. Refrigeration stations had to be set up at the ports of call along this route which are being touched by this ship and its sister, the Philippine Clipper. The first two of these stations were on Midway Island and Wake Island, and at present, equipment is being installed at Guam. The refrigeration requirement was not large at each point, on the order of five tons at 35 deg. F., but climatic conditions were exacting and no local facilities were at hand at all. Refrigerators are among the largest to be factory built in their particular manner. Constructed in the C. V. Hill factory in Trenton, N. J., each section was encased in galvanized iron on both sides, in order to withstand attacks by tropical insects. The basic construction is of wood, with a waterproofing layer under the sheathing. The insulation was of eight layers of Alfol (aluminum) in the crumpled form, occupying a space 4 in. thick. There was 20,000 sq. ft. used. This was specified not only for thermal reasons but on account of the light weight and for general economy. To complete the job the boxes were knocked down and shipped to their destinations. These boxes have three compartments each, designed for temperatures of 34 to 36 deg. F. Each has two 2-ton Brunswick compressors using methyl chloride in Bush finned coils, fed with thermal expansion valves.

R. H. O.

Tantalum.—A publication by the Fansteel Metallurgical Corporation begins with: "Tantalum—a rare metal with amazing properties of heat and electrical transference—with unbelievable resistance to the destructive forces of acid corrosion, oxidation and wear."

Tantalum, a metallic element, although only recently coming into prominence, is not a new metal. It was noted as an element in 1803 by Ekeberg, who gave it its name (derived from the mythical Tantalus) because he could not separate the element from its tightly locked compounds.

The successful production of tantalum is attended by many difficulties. The metal must be produced with a purity better than 99.9 per cent., its melting point is far higher than that of any available crucible which might hold it, and it can be dissolved in almost no reagent. Tantalum is made from the ore Tantalite, which is an iron-manganese Tantalate. This ore is found in limited quantities in many localities, but only in the desolate Pilbarra district of western Australia, 450 miles from the nearest railway, is Tantalite found in sufficient concentration to permit operations on a scale which will take care of present industrial requirements.

Tantalum metal cannot be produced by a simple smelting process. The several steps are chemical in nature and, to assure the production of satisfactory metal, each step must be controlled with scientific exactness. Errors made in the earlier stages cannot be rectified later in the process. Even if the pure tantalum powder is obtained, the remaining steps are rendered difficult because of the metal's extremely high melting point. The metal is not fused but is produced by pressing, sintering at extremely high temperatures, hammering and swaging. As tantalum combines readily with all common gases at elevated temperatures, subsequent operations, such as swaging and wire drawing, rolling sheet, and drawing seamless tubing are done cold, with vacuum annealing at suitable stages.

Tantalum is a heavy metal with a density of 16.6, about twice that of steel. A cubic foot of tantalum weighs 1034 pounds. Unannealed tantalum sheet is comparable to cold rolled steel. It is comparable to nickel in its ability to be drawn, stamped, or formed into complicated shapes.

Tantalum is inert to all acids except hydrofluoric, fuming sulphuric, boiling concentrated sulphuric and boiling concentrated phosphoric. Certain limitations attend its use with alkalies. Nearly all corrosive gases, whether wet or dry, are without effect upon tantalum at temperatures below 150° C.

The industrial applications of tantalum are many and varied. The chemical industry finds use for it in the production of highly corrosive acid substances. Tantalum is particularly suitable for the nozzles and spinnerettes in cellulose and rayon manufacture. The characteristics of tantalum make it valuable for electronic tube parts, such as cathodes, getter cups, grids, plates and support rods.

Tantalum carbide is extremely hard and has a melting point of approximately 4000° C. An important ingredient of hard-alloy cutting tools, drawing discs and wear resisting parts, tantalum carbide imparts to carbide tools the non-cratering characteristic which makes practical the machining of steel.

C.

Electricity Purifies Milk by New Process.—(*Electrical World*, Vol. 106, No. 13.) At the plants of the Foremost Dairies, Inc., Birmingham, Jacksonville and Atlanta, electricity is being used to purify milk without cooking it. Current at 220 volts is passed through the milk and a temperature of 165 deg. maintained for 20 seconds. This contrasts sharply with the old method of maintaining a temperature of 145 deg. for 30 minutes. Milk pasteurized by the older process developed a cooked taste. This has been overcome by the new method.

R. H. O.

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